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Topical Review

Multi-source errors evaluation of machine tools: from research gaps to methodologies and applications

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Abstract

Multi-source errors, as critical obstacles limiting the accuracy retention and machining performance of machine tools, hold fundamental and strategic significance for achieving high-precision, high-efficiency, and high-reliability machining in modern manufacturing systems. However, these errors typically exhibit complex characteristics such as strong coupling, time-variance, and nonlinearity, which challenge traditional methods of error identification, modeling, and compensation in terms of adaptability, real-time capability, and integration. Therefore, it is imperative to establish a systematic and intelligent multi-source error control framework. Firstly, this work systematically reviews typical error sources and their evolution mechanisms, evaluates multi-scale detection technologies including laser interferometry, double ball-bar systems, multi-sensor fusion, and vision-based systems, and constructs an intelligent error identification and evaluation framework. Next, it reviews classical

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modeling methods such as homogeneous transformation matrices, screw theory, thermal equilibrium models, finite element analysis, and modal analysis, compares physical modeling, data-driven, and hybrid modeling strategies, and develops an integrated multi-source error modeling architecture centered on digital twin technology and artificial intelligence. Furthermore, key technologies, including geometric error mapping and real-time compensation, online thermal error prediction and active temperature control, dynamic error suppression, and adaptive control, are summarized. A multi-level integrated error compensation architecture is proposed by combining physical models, data models, and cyber-physical synchronization. This architecture encompasses core processes such as error traceability and decoupling, dynamic prediction, real-time compensation, and closed-loop optimization, emphasizing engineering implementation mechanisms based on cyber-physical collaboration, multi-physics coupling, and multi-scale fusion, thereby effectively enhancing accuracy stability and control robustness under complex operating conditions. Finally, frontier challenges such as constructing high-fidelity coupled models from heterogeneous multi-source data, edge–cloud collaborative control, and cross-platform interoperability are discussed. The application prospects of multi-source error evaluation are also envisioned, providing theoretical foundations and technical support for the precise management and optimization of the entire lifecycle accuracy of machine tools.

Keywords: multi-source error evaluation, error coupling, integrated error modeling, traceability and decoupling, hybrid prediction, closed-loop error compensation

1. Introduction

Machine tools, as core equipment in manufacturing, have their precision directly determining product quality, production efficiency, equipment operational stability, and overall system performance. Various errors not only reduce machining accuracy but also lead to increased energy consumption, resource waste, and prolonged production cycles, severely limiting system optimization and competitiveness. Although CNC and automation technologies have made significant advances, challenges remain in dynamic error capture, real-time monitoring, and fast computation. In particular, the lack of efficient multi-source error testing and evaluation technologies has become a major bottleneck for maintaining machine tool accuracy. Therefore, achieving efficient error identification, modeling, and suppression has become a key research focus in the intelligent manufacturing and high-end equipment sectors.

To comprehensively understand current research trends, a bibliometric analysis was conducted using the Web of Science database (as of 31 December 2024), focusing on the keywords “machine tool”, “error”, and “precision”, and using VOSviewer software. The results show that current research hotspots concentrate on areas such as error classification, measurement technologies, modeling methods, error source tracing and decoupling analysis, prediction mechanisms, and compensation control strategies, as shown in Figure 1.

Further literature retrieval and clustering analysis based on “machine tool error” and its subfields reveal that studies mainly focus on improving error identification and detection technologies, evolving modeling theories, and optimizing compensation strategies, as shown in Figure 2.

Machine tool errors have complex causes, including geometric errors, thermal errors, cutting force errors, dynamic errors, mechanical wear, and control system errors, as shown in Figure 3. Traditional evaluation methods, primarily based on experimental analysis and theoretical modeling, such as laser interferometry[1] and ball-bar tests[2], perform well under static conditions but fail to meet the requirements of multi-axis linkage, complex working conditions, and dynamic responses[3]. Theoretical modeling methods, such as cutting force models[4,5] and finite element analysis[6,7], offer scalability and low cost[8,9], but under high-speed dynamic machining environments, they often lack modeling accuracy and suffer from response delays, making it difficult to meet the demands for real-time error identification and prediction[10]. Traditional compensation strategies rely on static pre-calibration and offline data, lacking the rapid adaptability to thermal loads, cutting forces, and servo dynamics.

Multi-source errors originate from dynamic interactions among various internal and external factors during machining. Geometric errors define the baseline static precision, thermal expansion causes time-varying structural deformation, cutting forces induce tool deflection and machine compliance variation, and dynamic vibrations create transient displacement errors. In high-speed multi-axis machining, these errors exhibit significant nonlinear coupling and complex accumulation, severely limiting tolerance precision in extreme manufacturing processes, particularly in fields such as aero-engine impeller manufacturing, semiconductor wafer stepping, precision optics, and large mold machining.

To overcome the limitations of traditional methods, emerging technologies such as artificial intelligence (AI), digital twin (DT), and big data analytics have enhanced adaptive error management under complex multi-axis machining conditions through virtual-physical fusion, dynamic evolution, and intelligent optimization capabilities[12−20]. Digital twin technologies achieve high-fidelity mapping and dynamic updates of error states, enabling dynamic tracking and multi-dimensional coupling modeling of errors[21−25], and supporting integrated

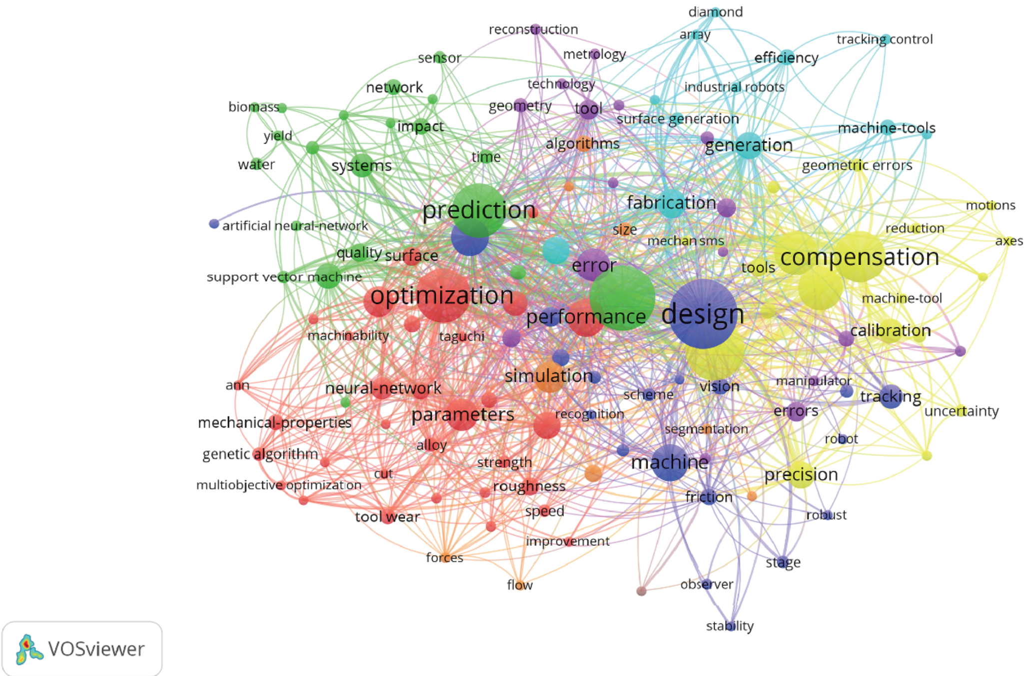

Figure 1. Comprehensive bibliometric analysis of machine tool errors.

state perception, mechanistic modeling, and closed-loop control[26,27]. AI technologies, based on deep learning and knowledge graphs, enhance robustness in error identification and intelligence in prediction[28]. Current research is gradually shifting from qualitative analysis of single error types to systemic studies focusing on multi-source information fusion, cross-scale modeling, and real-time closed-loop control[29−31], aiming for efficient perception, modeling, evaluation, and control of multi-source errors[32,33], as illustrated in Figure 4. This has become a central issue for improving machine tool precision retention, particularly in multi-axis linkage and intelligent manufacturing scenarios.

Compared with traditional approaches, emerging technologies demonstrate significant advantages across multiple stages, including error identification, modeling, source tracing and decoupling, and compensation, as shown in Table 1.

Cutting-edge technological breakthroughs and advances in real-time adaptive control support effective detection and control across various error types, particularly excelling in real-time monitoring, error prediction, compensation, and optimization, further underscoring the need for system-level solutions, as shown in Table 2.

To address these needs, this paper adopts a “divide and conquer” and “prioritize the major without neglecting the minor” strategy, and conducts a systematic review of methodologies and applications for evaluating multi-source errors in machine tools, as shown in Figure 5. Compared with recent reviews[50,51], this work adopts a multi-level, closed-loop system perspective and innovatively integrates multi-source error identification, integrated error modeling, source tracing and decoupling, adaptive prediction, and intelligent compensation. It constructs a closed-loop intelligent control system of “perception–modeling–analysis–feedback optimization”, where the perception layer captures heterogeneous multi-source information, such as thermal, cutting force, and vibration data in real time, and employs data preprocessing, fusion, and feature extraction to supply high-quality inputs for modeling. The modeling layer uses multibody dynamics, finite element methods, and machine learning to build physics-driven, data-driven, and hybrid error models, achieving digital representation and evolution of error behaviors. The analysis layer conducts error state evaluation and trend prediction based on real-time monitoring and error evolution, generating optimal control strategies. The feedback optimization layer dynamically adjusts machine tool operation status and performance through servo tuning and online compensation, forming a cyber-physical collaborative closed-loop control system. This framework realizes full-chain intelligent management of multi-source errors. It conducts in-depth analysis of nonlinear error coupling mechanisms, emphasizes digital twin–AI integrated modeling paradigms, explores key challenges of real-time adaptive modeling, and establishes quantitative evaluation metrics to enhance comparability and industrial adaptability. It also summarizes error tracing and decoupling methods, high-fidelity prediction, and self-optimized compensation under high-dynamic and complex conditions. Finally, it envisions an autonomous adaptive error management system for intelligent manufacturing, promoting systematic, real-time, and standardized advancement of machine tool precision retention.

The remaining part of this article is structured as follows: chapter 2 reviews the sources, classification, and influencing mechanisms of errors; chapter 3 compares the advantages and disadvantages of traditional and intelligent systems in

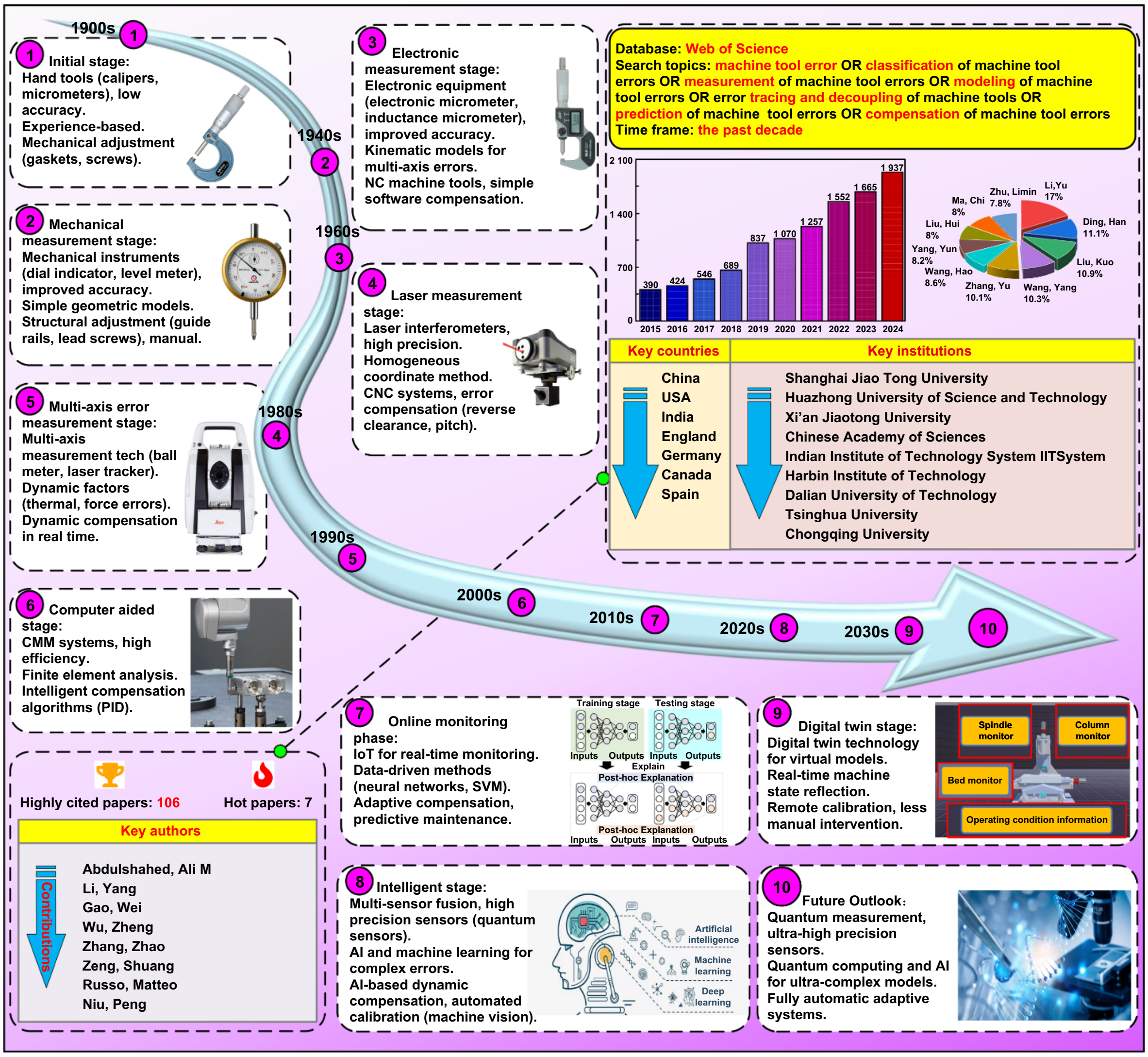

Figure 2. The development process of the core research on machine tool errors.

error identification and assessment; chapter 4 focuses on error modeling methods, covering physical, data, and hybrid modeling; chapter 5 explores error traceability and decoupling strategies; chapter 6 reviews advanced error prediction technologies; chapter 7 summarizes real-time error compensation strategies and constructs a multi-level integrated compensation system; chapter 8 summarizes the entire article; chapter 9 looks forward to future research directions.

2. Source and classification

2.1. Geometric error

Geometric errors are critical factors affecting machine tool machining accuracy and part quality, widely present in various precision manufacturing scenarios. They stem from multiple mechanisms, including machine structure, thermal deformation, control system errors, and component wear, directly leading to deviations in machining dimensions, shape, and surface quality. Therefore, a deep understanding of the formation mechanisms and influence paths of geometric errors is essential for improving manufacturing precision and machine tool performance.

Firstly, one of the main sources of geometric errors is the inherent inaccuracy in machine tool mechanical structures, especially positioning errors of key components such as the bed, column, and spindle during manufacturing and assembly[55]. Even with high-precision manufacturing processes, structural errors are inevitable due to material property differences and manufacturing tolerances. For instance, misalignment of guideways often causes linear motion deviations, affecting the accuracy of machining paths[56].

Thermal effects are also a major cause of geometric errors. During operation, friction, cutting heat, and motor heating lead to expansion or contraction of key components, resulting in structural deformation and posture shifts[30]. For example, spindle elongation due to thermal expansion can cause tool deviation from the programmed path, thereby affecting machining accuracy[57]. ISO 230 standards propose that geometric accuracy evaluation should include indicators

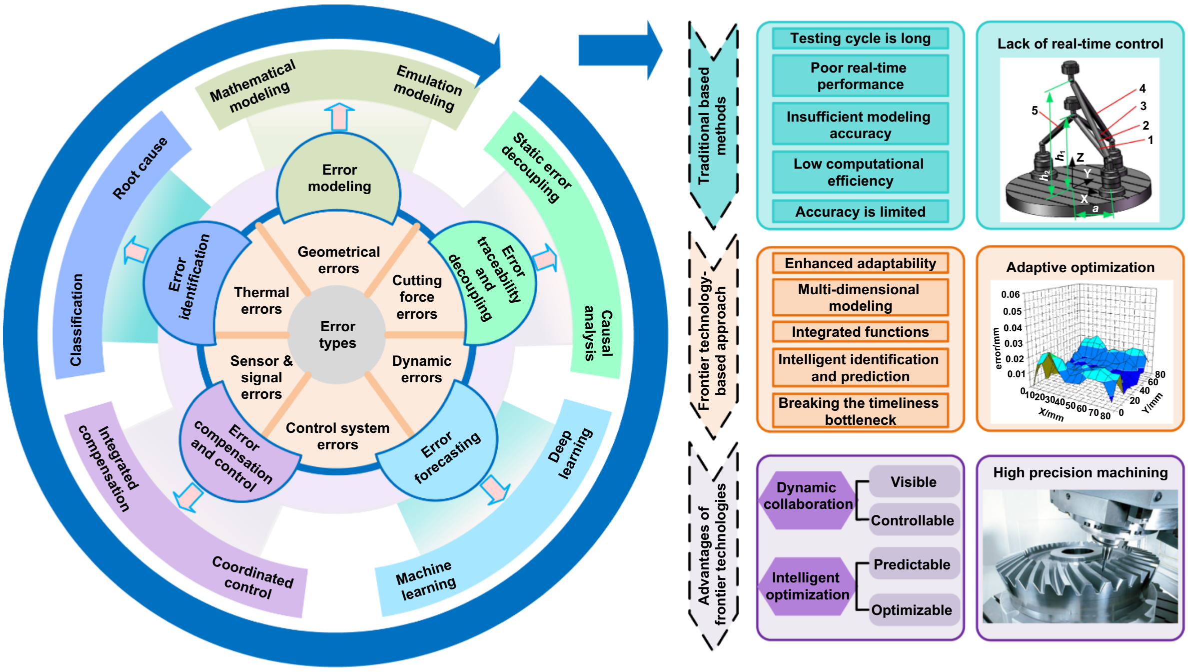

Figure 3. Main contents and methods of multi-source error research on machine tools. Reprinted from[11], Copyright (2013), with permission from Elsevier.

such as positioning error, straightness error, and angular error[58]. As thermal errors dynamically evolve with load and environmental changes, obtaining accurate patterns of their variation becomes the core challenge in thermal characteristic evaluation and compensation design.

As usage time increases, wear of machine tool components also induces geometric errors. For instance, prolonged operation of actuating components like bearings, lead screws, and gears results in increased backlash and reduced motion accuracy, leading to repeated positioning errors and trajectory deviation[59]. Particularly under prolonged maintenance-free or heavy-load conditions, wear errors accumulate, significantly affecting the machine tool's long-term precision retention capability[60,61].

Additionally, the CNC control system itself may be a source of geometric errors. Although CNC aims to achieve high precision and repeatability, errors in control algorithms, interpolation strategies, or feedback signals can still cause deviations in machining paths[62]. For example, improperly set position loop gain or signal distortion from encoders can more easily lead to contour errors in high-speed machining scenarios[63].

Geometric errors can have far-reaching impacts on part quality and functionality. For instance, squareness errors in rotary axes of five-axis machine tools can result in nonlinear contour deviations, affecting assembly accuracy and operational reliability[64]. Furthermore, geometric errors alter cutting force distribution, leading to deteriorated surface roughness, increased tool wear, and residual stress concentration[65,66]. In high-end manufacturing fields such as aerospace and medical devices, even tiny errors can cause systemic failure or safety risks[67–69].

To effectively suppress geometric errors, machine tool manufacturers continuously optimize structural design and assembly processes by adopting technologies such as preloaded bearings, tensioned lead screws, and high-stiffness guideways to reduce looseness and motion gaps[60]. At the same time, CNC systems incorporate error compensation algorithms to correct tool paths in real time based on modeling results, such as adjusting control parameters through online learning mechanisms or optimizing servo system response via multi-machine data sharing[70,71]. In addition, regular maintenance and precision calibration remain essential for long-term accuracy assurance[68].

In recent years, real-time monitoring systems integrating sensors and feedback control have become increasingly popular. These systems can dynamically perceive machine tool operating status and adaptively correct machining paths, thereby improving overall system accuracy and robustness. In the future, geometric error control will continue to focus on intelligent compensation technologies, advanced material applications, and innovations in machine tool structure to achieve higher quality and greater manufacturing consistency.

2.2. *Thermal error*

In modern intelligent manufacturing systems, controlling thermal errors in machine tools has become a research hotspot in the field of precision machining. As one of the primary factors affecting machining accuracy and product quality, thermal deformation not only degrades surface quality but may also cause instability in the machining system. Studies show that thermal errors account for as much as 60% to 75% of total machining error[72]. Therefore, minimizing thermal sources during the design phase and suppressing thermally induced errors during operation are key strategies to ensure precision retention and stability in precision machine tools.

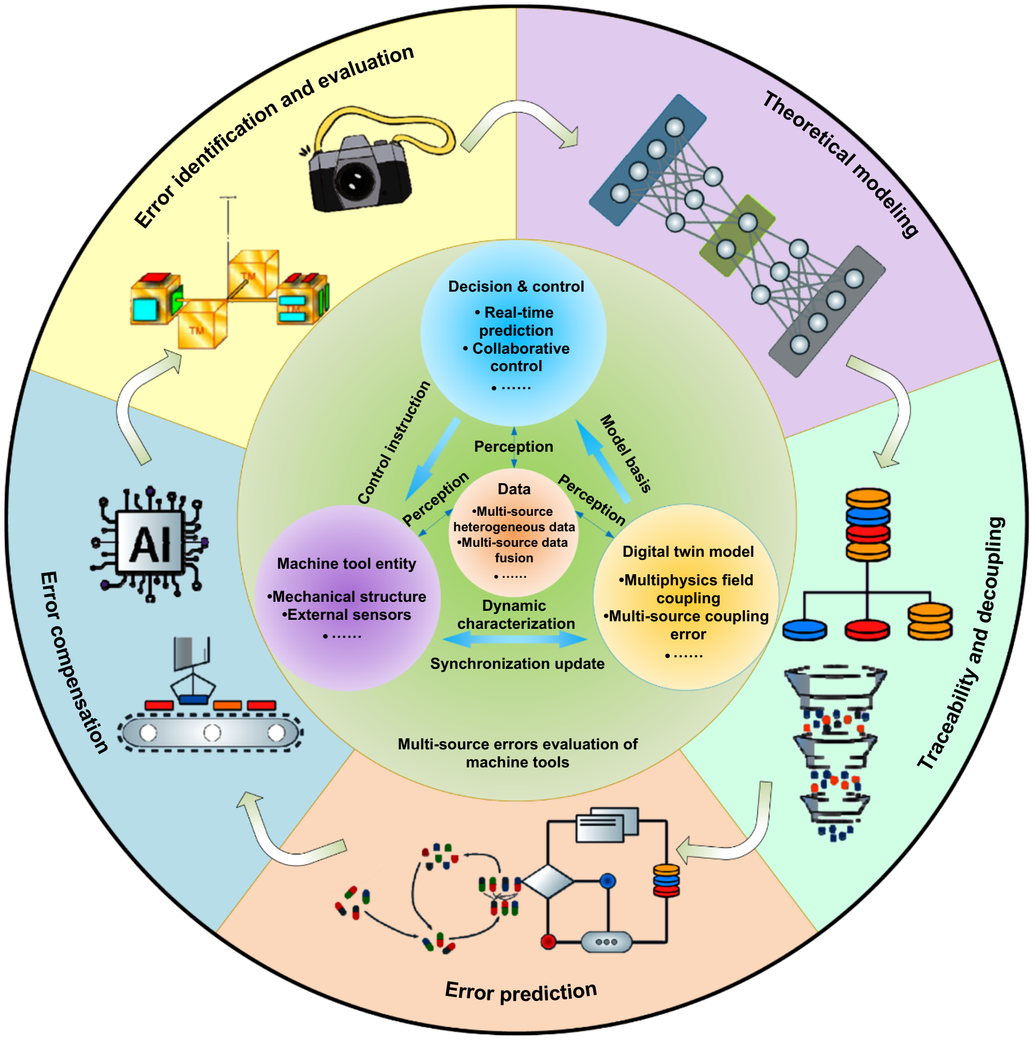

Figure 4. Methodology and application in multi-source error evaluation of machine tools.

2.2.1. The main cause of thermal error. J. Bryan's pioneering work first systematically revealed the impact of machine tool thermal deformation on machining accuracy and identified spindle thermal expansion as one of the main causes of geometric errors[73]. Based on the nature of heat sources, thermal errors in machine tools mainly originate from external heat sources, the spindle, the feed system, and cutting heat. These act on machine tool structures via conduction, convection, and radiation, leading to uneven temperature field distribution and subsequent changes in the relative position between tool and workpiece, as shown in Figure 6.

As shown in Figure 6(a), the fluctuation of environmental temperature causes changes in the overall thermal field of the machine tool structure, resulting in thermal deformation of the structural components. This is the main external factor affecting the thermal error. Therefore, precision machine tools typically operate in temperature-controlled environments. Some studies have built mapping models between environmental temperature variation and thermal deformation using thermal error transfer functions and frequency-domain analysis[76], or established multivariate regression-based forecasting frameworks combining Fourier series and time-series modeling[77]. Moreover, thermal radiation from hydraulic stations and electrical cabinets can cause local heating, which can be effectively isolated using thermal insulation panels[78].

As shown in Figure 6(b), as the core component of the cutting system, the main spindle of the machine tool generates a large amount of heat when operating at high speed, which is prone to causing thermal elongation and thereby lead to processing errors. Research mainly focuses on material selection and cooling system optimization to suppress spindle thermal errors. New materials like carbon fiber reinforced polymer (CFRP) and glass ceramics offer excellent thermal stability, effectively reducing thermal deformation of spindles[79,80], although they may compromise spindle stiffness and dynamic characteristics. Therefore, cooling system design has become the mainstream strategy for controlling spindle thermal error[81].

Low-load spindles often use air convection cooling[82], while high-speed, high-torque spindles tend to adopt water or oil cooling systems. Recently, researchers have improved heat exchange efficiency by optimizing the cross-section shape, curvature, and roughness of cooling channels[83]. Model-driven cooling control strategies based on ambient temperature

Table 1. Comparison between traditional methods and intelligent systems in machine tool error evaluation process.

Research content	Key indicators	Characteristics of traditional methods	Advantages of intelligent systems	References
Error identification	Identification accuracy, identification speed	Limited accuracy, delayed response	Real-time identification, high accuracy, strong dynamic response	[32]
Error modeling	Model fidelity, visualization	Simplified modeling, lack of intuitiveness	Driven by real-time feedback, high modeling accuracy, supports visual analysis	[33–35]
Error traceability and decoupling	Traceability, decoupling accuracy	Static analysis, experience-dependent	Supports dynamic tracing under complex conditions with high-accuracy decoupling	[36–39]
Error prediction	Prediction accuracy, prediction speed	Delayed prediction, poor dynamic adaptability	Real-time prediction, strong interactivity, good adaptability	[40–42]
Error compensation	Compensation reliability, compensation accuracy	Simple mechanism, poor adaptability	Closed-loop feedback compensation, strong adaptability, high precision	[43,44]

Table 2. Comparison between traditional methods and intelligent systems for typical error types.

Error type	Characteristics of traditional methods	Advantages of intelligent systems	References
Geometric error	Offline measurement after processing, limited error source analysis	Digital twin comparison analysis of geometric error mechanisms, higher modeling accuracy	[44]
Thermal error	Only analyzes the single heat source effect, difficult to measure	Multi-source heat simulation and feedback compensation, improved thermal control capability	[37–40,45]
Cutting force error	Static measurement and compensation, difficult to handle dynamic conditions	Real-time monitoring and modeling, improved processing adaptability and compensation accuracy	[46,47]
Dynamic error	Dynamic signals analyzed after experiments, no real-time adjustment	Dynamic monitoring and real-time compensation, improved processing stability and quality	[42,48,49]

tracking can further improve response speed and accuracy of the cooling system[84–86].

As shown in Figure 6(c), the components of the feed system, such as the servo motor, lead screw, guide rail, and bearing generate heat during operation, causing thermal deformation of the structure and thereby affecting the positioning accuracy of the tool. Air cooling and liquid cooling are common thermal management methods for lead screws; the former suits high-speed, low-load scenarios, while the latter is better for high-load machining[87,88]. Thermal error from lead screws can be effectively managed using heat transfer models combined with adaptive fuzzy PID control strategies[89]. Compared to spindle thermal management, research on feed system thermal errors is relatively lacking. More work is needed to investigate the relationship between thermal source characteristics and structural precision.

As shown in Figure 6(d), the high-speed friction in the cutting zone generates a large amount of heat, which affects the stability of the process system and the lifespan of the cutting tool. Cutting heat management strategies include external cooling, internal cooling, and hybrid cooling[90,91]. External cooling sprays coolant or low-temperature gas to suppress temperature rise in the machining zone[92,93]; internal cooling channels within the tool directly cool the cutting point[94–96]; hybrid cooling combines the strengths of both, particularly suitable for ultraprecision machining scenarios, improving both cooling efficiency and tool stability[97,98].

2.2.2. Types and mechanisms of thermal errors. Thermal errors can be categorized into four types: thermal deformation errors (structural deformation caused by a single heat

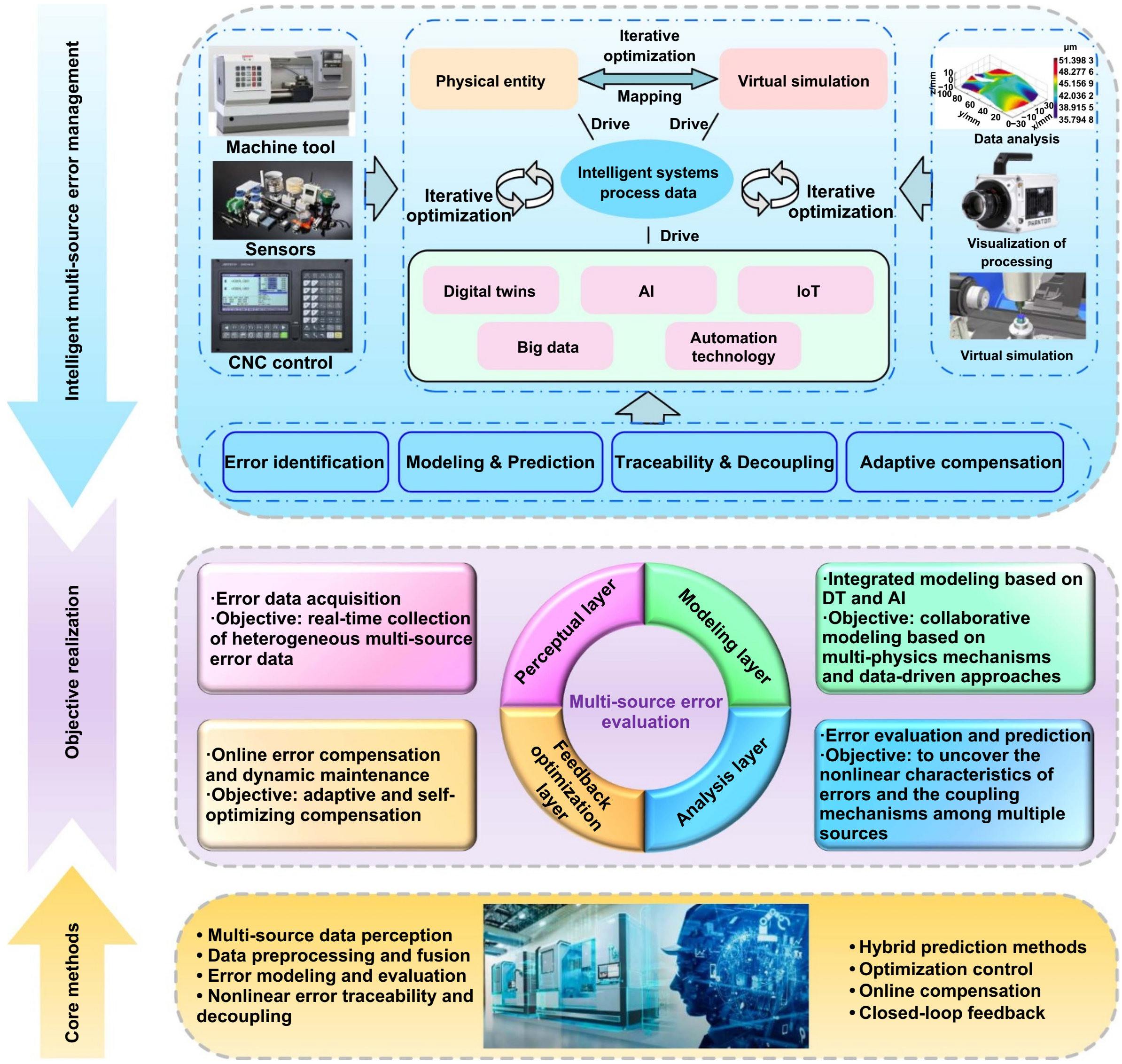

Figure 5. Application of intelligent system in machine tool errors[52–54].

source), thermal drift errors (system-wide offset due to asymmetric cooling), thermal disturbance errors (random disturbances from external radiation or internal coupled heat sources), and thermal stability errors (instabilities due to long-term thermal equilibrium or self-excited oscillation)[99–101].

Thermal errors essentially arise from the uneven distribution of internal heat sources and the time-varying nature of thermal properties of materials. Under thermomechanical coupling, the machine tool's internal temperature and stress fields become non-uniform, leading to form and positional errors such as guideway displacement and spindle runout[102–104]. Thus, accurately establishing the mapping between temperature fields and thermal deformation, and building efficient thermal error prediction models, are central to thermal error control.

Currently, relevant research mainly focuses on two directions: first, based on thermal characteristic modeling and sensitivity analysis, to identify key heat sources and quantify thermal influence paths[105]; second, to construct multi-physics coupled models for high-precision simulation of machine tool thermal behavior and to develop active thermal error control strategies[89–106]. Multiphysics modeling is key to achieving submicron- and even nanometer-level precision control[107], and remains a technical bottleneck in high-end equipment manufacturing. A typical method is to use finite element analysis (FEA) to build coupled models of structural temperature and thermal deformation fields, and integrate sensor arrays to achieve real-time thermal-displacement monitoring and dynamic mapping[108].

On this basis, researchers have introduced intelligent algorithms such as Model Predictive Control (MPC), Adaptive Control, and Reinforcement Learning to enhance the precision, response speed, and robustness of online thermal error compensation[109,110]. Using digital twin platforms, it is also

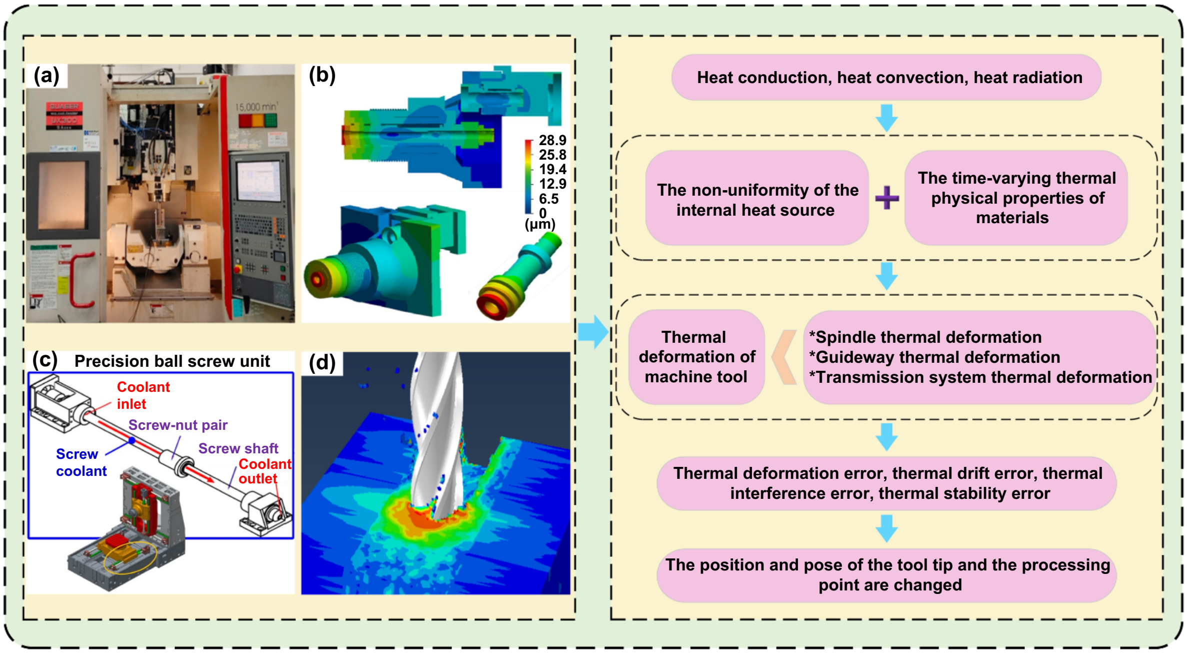

Figure 6. Schematic diagram of the influence of heat on the machine tool. (a) External heat source. Adapted from[74], with permission from Springer Nature. (b) Spindle[74]. Adapted from[75], with permission from Springer Nature. (c) Feed system[75]. (d) Cutting heat.

possible to implement thermal design simulation, strategy testing, and parameter optimization in virtual space, forming a "modeling–simulation–feedback–optimization" closed-loop system to support thermal error management across the machine tool lifecycle[111].

2.3. Cutting force error

Cutting force, a direct manifestation of the interaction between tool and workpiece, not only determines material removal efficiency but also significantly affects dimensional accuracy, form error, and surface quality. In scenarios with complex structures or insufficient rigidity, it can easily induce machining errors. Cutting forces are typically divided into tangential, radial, and axial components: tangential force influences shear deformation and energy consumption, and determines tool stability; radial force acts perpendicular to the machining axis, causing tool deflection and structural vibration, reducing dimensional accuracy and surface integrity; axial force acts along the spindle direction, significantly impacting tool wear and system rigidity balance[112].

In actual machining, cutting force is influenced by multiple coupled factors such as workpiece material, tool geometry and material, and cutting parameters (speed, feed, depth)[113]. For example, harder or low-thermal-conductivity materials often result in higher cutting forces, exacerbating tool deflection and thermal deformation; tool wear alters cutting force distribution, further amplifying errors[114–116]. When the tool overhang is long or system rigidity is insufficient, deflection due to cutting force becomes more pronounced, especially in thin-walled structures or micro-machining, where dimensional deviations are non-negligible[117]. In thin-wall aerospace component milling, cutting force fluctuations cause static tool deflection and dynamic workpiece vibration, leading to wall thickness variation and roundness deviation that often exceed assembly tolerances[118].

Moreover, instabilities during cutting—such as self-excited chatter—are a major source of error accumulation. Chatter is characterized by "positive feedback"; once initiated, it causes persistent cutting force fluctuations, surface degradation, and shortened tool life[119,120]. Simultaneously, cutting friction and plastic deformation generate significant heat, causing localized thermal expansion of the tool and machine, inducing positioning errors—especially pronounced in high-speed machining[121]. The superposition of thermal deformation and force-induced deflection increases thermo-mechanical coupling instability, complicating error modeling and control.

Therefore, effective control strategies for cutting force errors should include multi-level coordinated measures such as cutting parameter optimization, tool design rationalization, enhancement of system rigidity, and thermo-mechanical coupling compensation mechanisms, to ensure machining stability and consistency at high precision.

2.4. Dynamic error

Dynamic errors are key factors affecting quality and consistency in high-speed precision machining, primarily originating from structural vibration, inertia effects, servo lag, and system compliance. Essentially, these are transient deviations arising in machining systems under rapid responses and coordinated multi-axis movements, exhibiting strong nonlinearity and time-varying characteristics.

Among them, structural vibration and chatter are particularly prominent. The former often results from uneven motion of components or external disturbances; if not effectively suppressed, the disturbance energy propagates through the structure to the tool and workpiece, causing dimensional fluctuations and increased surface roughness. The latter, a typical self-excited vibration phenomenon, arises from unstable coupling between cutting and structural dynamics, usually manifesting as sharp noise and characteristic surface waviness, which not only damages surface integrity but may induce local stress concentration and reduce fatigue life[122,123].

Machine tool structural rigidity plays a crucial role in dynamic error control. If the bed, spindle, or connecting parts lack rigidity, deformation and oscillation occur, exacerbating tool deflection and unstable machining[124]. Vibration-induced alternating loads also accelerate tool wear, shorten tool life, and further affect machining efficiency and product accuracy[125,126].

Additionally, the material properties of the workpiece and tool significantly influence chatter sensitivity. Softer materials are prone to inducing vibration during machining and require optimized cutting parameters for suppression[122]; while harder materials exhibit better resistance to vibration but heightened sensitivity to tool wear, demanding tools with high rigidity and wear resistance[123–127].

Servo tracking error is a critical component of dynamic errors. The instantaneous deviation between the commanded trajectory and the actual position directly affects contour accuracy, especially in high-speed multi-axis machining scenarios, where it is particularly sensitive. This type of error is significantly influenced by the servo system bandwidth, control parameter matching, and synchronization performance among axes. Recent studies have emphasized the importance of optimizing servo-dynamic matching. For example, Ramesh et al.[128] pointed out that inconsistent responses among servo axes are the fundamental cause of endpoint errors; Kong et al.[129] proposed a commercial servo parameter tuning method that requires no secondary development to achieve matching optimization in five-axis systems; Guan et al.[130] further established a unified Servo Dynamic Matching Degree (SDMD) index system to quantitatively describe the dynamic response consistency between translational and rotational axes, which is used for collaborative optimization design in complex five-axis systems.

In terms of control strategies, intelligent servo control and active vibration suppression systems are widely used. Sensor-based state monitoring combined with damper feedback enables real-time vibration identification and suppression; variable-speed drives and adaptive control strategies can adjust spindle speed dynamically to reduce chatter risk. With improvements in sensor integration and signal processing capabilities, dynamic prediction and preventive control have become key to enhancing system stability and product consistency[125].

In summary, effectively suppressing dynamic errors requires full-chain coordinated optimization ranging from structural design and material selection to servo control. Systematic dynamic management not only improves machining accuracy and surface quality but also extends tool life and reduces manufacturing costs.

2.5. Other sources of error

Beyond the main error types mentioned, machine tool operating accuracy is also affected by factors like wear, backlash, and control system errors[131]. Although these latent errors may not manifest early in machining, they gradually accumulate over time and pose significant threats to equipment reliability and manufacturing consistency.

Component wear is a primary contributor to long-term precision degradation. Wear caused by friction, fatigue, and environmental corrosion can change the dimensions and surface state of critical moving parts, reducing dynamic response performance[132]. Regular maintenance and timely replacement of worn components can effectively suppress this type of error accumulation[133].

Backlash errors commonly occur in transmission structures such as lead screw and gear pairs; direction reversal processes introduce play that severely impacts positioning accuracy, particularly under high-frequency, high-speed reversal conditions[133]. Preloaded designs and backlash compensation algorithms are common solutions[134,135].

Control system errors stem from delays or deviations in sensing, processing, and execution stages. They may involve hardware faults (e.g., sensor failures, servo anomalies)[136], software bugs (e.g., trajectory planning errors, control logic mistakes)[137], and environmental interference (e.g., electromagnetic noise, power fluctuations, thermal drift)[134]. Enhancing control system robustness typically requires fault-tolerant control, calibration mechanisms, and electromagnetic shielding[138].

It is important to note that these error sources often interact rather than exist independently. For example, thermal expansion can affect geometric structure and system rigidity, which in turn alters dynamic response characteristics; heat generated by cutting force not only induces thermal deformation but also affects tool–workpiece contact; structural wear can increase backlash, further affecting servo feedback accuracy and inducing control errors[139,140].

Such multi-source coupling effects form complex nonlinear dynamic systems, making it difficult for traditional single-source error compensation methods to cope effectively. Recent studies have shown that constructing an integrated modeling framework for multi-source collaboration is key to achieving effective error control. For example, multi-source error identification and compensation strategies for five-axis flank milling scenarios[141], as well as thermal error prediction frameworks that integrate multi-source heterogeneous information, both demonstrate the urgent need for coupled modeling[142].

Future development should focus on system modeling and state recognition across multiple physics domains and time-space scales, combined with integrated health management systems to enable real-time monitoring and predictive control of error sources[143,144]. By integrating data-driven modeling with knowledge inference mechanisms, error management can evolve from 'static prevention' toward 'dynamic intervention', thus ensuring long-term precision retention in machine tool operation.

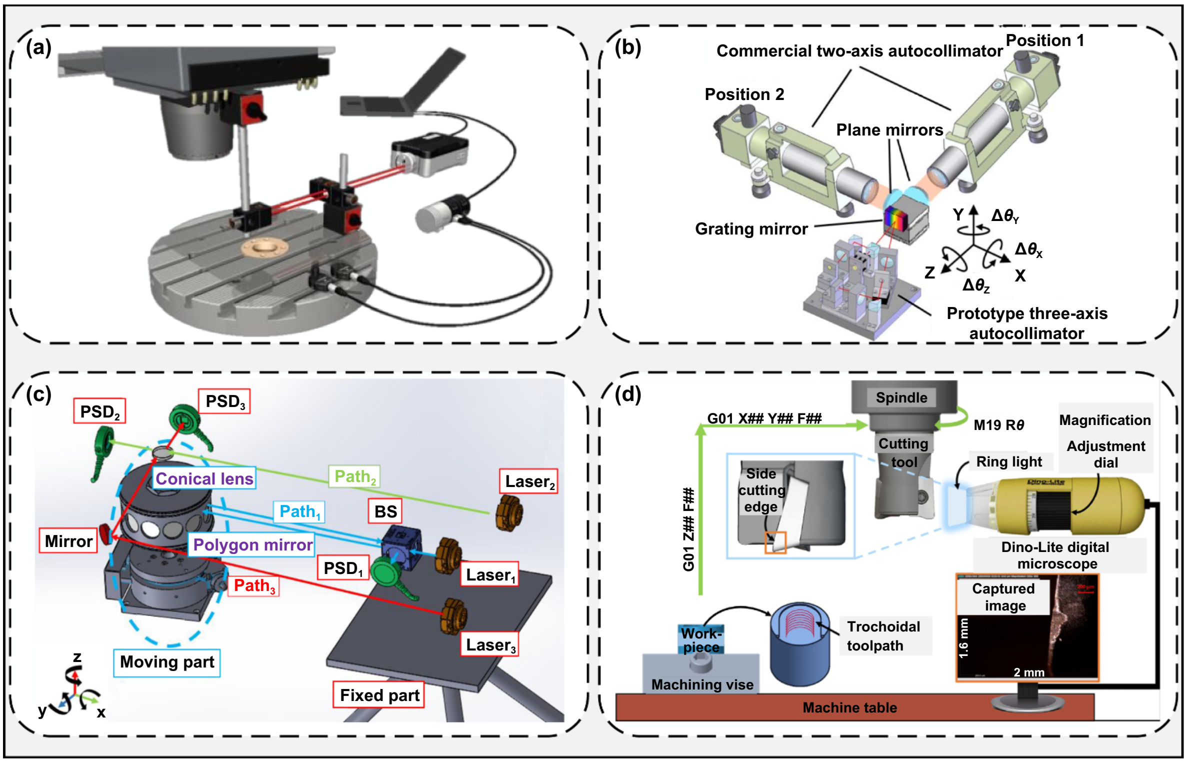

Figure 7. Summary of direct measurement methods. (a) Laser interferometric measurement. (b) Autocollimators and levels. Reprinted from[148], Copyright (2011), with permission from Elsevier. (c) Optical measurement methods. Reprinted from[149], Copyright (2020), with permission from Elsevier. (d) Machine vision inspection. Reprinted from[150], Copyright (2025), with permission from Elsevier.

3. Identification and evaluation

Rapid and accurate identification of machine tool errors is the foundation for high-precision manufacturing and intelligent compensation control. It directly influences error modeling, prediction, and dynamic adjustment capabilities. Current methods fall into traditional measurement and intelligent-system-based modern evaluation, leading toward a system targeting multi-source errors, high-resolution sensing, and dynamic feedback control to support high-quality modeling and compensation.

3.1. *Traditional methods*

Based on measurement mode, traditional methods include direct and indirect measurement[145]. Direct methods use specialized instruments to measure errors with high accuracy, but are often environment-dependent. Indirect methods infer error parameters from axis trajectories, workpiece geometry, or motion characteristics—suitable for dynamic error evaluation in complex conditions[146].

3.1.1. Direct measurement methods. Direct measurement methods refer to the independent measurement of individual error components (such as positioning error, straightness error, and angular error) of each machine tool axis using high-precision metrological instruments, in accordance with the ISO 230-1 standard[147]. These methods are typically applied during machine tool assembly, alignment, or final accuracy acceptance testing, offering high measurement resolution and traceability. Representative direct measurement techniques include laser interferometry, autocollimators and levels, optical metrology methods, and machine vision inspection, as illustrated in Figure 7.

As shown in Figure 7(a), laser interferometry is widely used to evaluate the linear displacement, straightness, perpendicularity, parallelism, and angular errors of machine tools[151,152]. A typical instrument is the laser interferometer, which leverages the coherence of laser light to detect changes in optical path length through interference fringe variations, thereby enabling highly precise error measurements. The measurement accuracy can typically reach the nanometer scale, making it suitable for high-precision static error detection and compensation under controlled environmental conditions[153,154]. To improve measurement efficiency, multi-degree-of-freedom (MDOF) laser interferometric systems have been extensively studied. These systems often employ multi-beam path splitting configurations or diffraction gratings to simultaneously measure errors across several degrees of freedom. For instance, Liu et al. developed a three-degree-of-freedom interferometric system; Hsieh proposed a six-degree-of-freedom heterodyne grating-based system; and Lu introduced a symmetric Littrow-configuration grating encoder. These systems have demonstrated high resolution and excellent measurement consistency in experimental validations[11,55,156]. Despite its outstanding accuracy, laser interferometry is highly sensitive to environmental disturbances and requires costly equipment, which limits its widespread industrial deployment.

As shown in Figure 7(b), the measurement of the attitude error is usually carried out using autocollimators and level gauges. The former can achieve high-precision measurement of the pitch and yaw angles by utilizing the principle of light beam reflection, while the latter detects the roll error based on the gravity reference[147]. In recent years, the development of triaxial angular measurement units—such as triaxial autocollimators and triaxial inclinometers—has significantly expanded the applicability of these techniques and enhanced their capability for simultaneous multi-degree-of-freedom error detection[148,157,158].

As shown in Figure 7(c), for the measurement of the six-degree-of-freedom error of the rotating axis, various optical measurement methods have been proposed in previous studies. Chen et al.[159] developed a photoelectric measurement system comprising a conical–polygonal mirror and three pairs of laser diodes with position-sensitive detectors (PSDs). By detecting the displacement of laser spots on the PSDs, this system enables non-contact measurement of the 6-DOF motion errors of rotating components. Subsequently, Liu et al.[149] proposed an optical measurement device based on a combination of a conical lens and a multi-facet mirror, which also allows for the simultaneous detection of all six error components of rotary axes.

Compared with linear axes, rotary axes pose greater challenges for precise and synchronous error measurement due to the inherent uncertainty of rotational motion and the strong coupling between error components. Accurate 6-DOF measurement of rotary axes therefore remains a critical and unresolved issue in the field of precision metrology.

As shown in Figure 7(d), machine vision is based on image acquisition and processing technology and is suitable for non-contact and high-speed automated detection tasks. It can achieve error feature recognition, visualization, and real-time tracking[160,161]. However, its recognition accuracy relies on extensive sample training, and it may face risks such as poor robustness and overfitting under complex working conditions[162,163].

Overall, the direct measurement method has the advantages of high measurement accuracy, clear decoupling of error terms, and traceable results. It is suitable for the standard detection and initial modeling of the geometric performance of machine tools. However, its adaptability in identifying the comprehensive errors of the entire machine tool or in dynamic working conditions at the production site is relatively limited.

3.1.2. Indirect measurement methods. Indirect measurement methods estimate various error sources by measuring the machine tool's response after executing specific trajectories or machining standard workpieces, combined with error model inversion. Their core advantage lies in not requiring sensors to be installed individually on each error source, enabling collaborative identification of system-level errors for the entire machine. These methods are suitable for rapid evaluation and error compensation modeling of machine tools during operation. Typical techniques include the diagonal test and step diagonal test, laser trackers, double ball-bar tests, R-test and its derived extended methods, as illustrated in Figure 8.

As shown in Figure 8(a), the diagonal test and the step diagonal test are among the earliest standardized methods[166]. They evaluate inter-axis errors and spatial straightness by moving along multiple diagonal paths and performing distance measurements[167,168], but they have difficulty capturing angular errors. In traditional diagonal tests for spatial error identification of machine tools, limited measurement trajectories often lead to ineffective decoupling of multi-axis geometric errors. To improve the completeness and observability of error identification, subsequent studies have proposed improved methods such as the three-face step method and the 13-line method. These approaches design spatial measurement trajectories with multiple directions and postures to simultaneously identify position-dependent geometric errors (PDGEs) and position-independent geometric errors (PIGEs), thereby significantly enhancing the modeling and evaluation capabilities for multi-source errors in machine tools[169,170].

As shown in Figure 8(b), the laser tracker, due to its high precision, wide range, and three-dimensional dynamic tracking capabilities, has been widely applied in the identification of spatial errors and geometric accuracy verification of high-end gantry machines and five-axis machining centers, becoming one of the important means to achieve high-precision modeling and error compensation[171,172]. A multilateration system constructed by multiple laser tracker stations can infer the three-dimensional position of target points through multi-source distance information without the need for angle measurements, enabling spatial error modeling and accuracy evaluation of machine tools[173]. To reduce equipment costs and installation complexity, pseudo-multilateration methods simulate multi-point measurement configurations by moving a single laser tracker, and have been widely used for on-site accuracy verification and error compensation modeling of large machine tools and complex equipment[174].

As shown in Figure 8(c), the DBB test is centered on detecting the deviation of circular interpolation trajectories. It features simple operation, rapid measurement, and strong environmental adaptability, and is widely used in the evaluation of geometric accuracy and dynamic performance of CNC machines[175]. It is particularly suitable for the needs of industries such as automotive mold manufacturing[176]. The DBB system records the actual deviations of the machine tool during two-axis coordinated circular motion via a ball-bar device, and then derives key performance parameters such as perpendicularity, backlash, and servo gain[58]. Although the coverage of the ball-bar system is limited, it is suitable for rapid on-site inspection[177,178]. Moreover, the combined application of DBB and laser interferometry integrates the advantages of both, enhancing the comprehensiveness and accuracy of rotary and linear axis error detection, and is often used for calibrating position-independent geometric errors (PIGEs) in high-end equipment[179].

In recent years, to improve measurement adaptability and identification capability, load double ball-bar (LDBB) systems

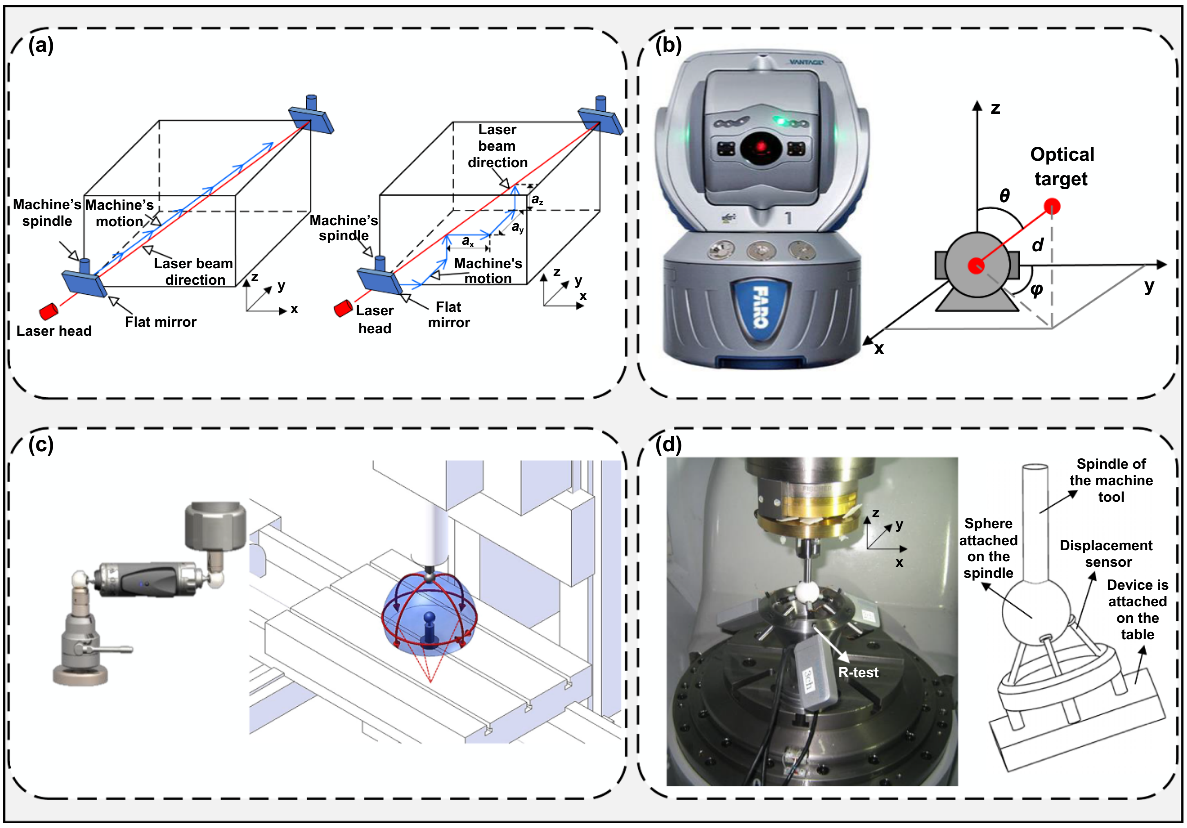

Figure 8. Summary of indirect measurement methods. (a) Diagonal test and step diagonal test. Reprinted from[164], Copyright (2010), with permission from Elsevier. (b) Laser tracker. (c) Double ball-bar test. (d) R-test measurement. Reprinted from[165], Copyright (2012), with permission from Elsevier.

with preload functions have been developed to simulate load conditions during machining, thereby identifying static stiffness responses and their effects on trajectory accuracy[180,181].

For the identification of rotary axis errors in five-axis CNC systems, researchers have proposed multi-path combination testing methods and two-step testing methods. By varying bar length, installation angle, and measurement path design, these methods achieve stepwise separation and modeling of position-dependent geometric errors (PDGEs) and position-independent geometric errors (PIGEs) of rotary axes, effectively addressing challenges such as error coupling and installation deviations[182,183].

As shown in Figure 8(d), to overcome the limitations of the ball-stick test in measuring spatial errors, Weikert[184] proposed the R-test instrument and method. This system comprises a probe and a standard ball, using multiple non-contact displacement sensors to detect minute displacements of the ball center in the X, Y, and Z directions in real time, thereby enabling comprehensive detection of spatial errors in machine tools. This approach effectively identifies spatial and positioning errors in five-axis machining centers, enhancing the accuracy of error modeling[185–188].

To overcome the precision bottlenecks of traditional three-axis testing in multi-axis systems, researchers have further developed advanced testing strategies such as the DBB system and the Multiple Test Arbors (MTA) method. For instance, Lei et al.[189] proposed a DBB method capable of evaluating dynamic errors of rotary axes in five-axis CNC machines and assisting in servo parameter tuning. Li et al.[190] developed the MTA strategy, which enables efficient calibration under complex postures through coordinated measurement with multiple arbors and virtual TCP constraints. This approach addresses core issues in hybrid machine calibration, such as complex equipment setup, difficulty in posture measurement, and tool change accuracy degradation, significantly improving the spatial calibration accuracy and modeling reliability of multi-axis systems.

3.1.3. Integration of direct and indirect methods.

Direct measurement methods and indirect measurement methods each have their own advantages in terms of application scenarios and technical characteristics. In actual manufacturing, the two are often used in a complementary and cooperative manner. The former is suitable for high-precision static error calibration and modeling initialization, while the latter is more appropriate for state evaluation and error tracking during dynamic operation. For example, ball-bar testing is widely applied in automotive mold production lines, enabling rapid assessment of roundness errors and servo tuning quality in multi-axis CNC systems, without the need to disassemble the machine tool for real-time adjustment. The R-test technique can characterize spatial errors in five-axis machining centers, allowing comprehensive evaluation of volumetric accuracy in the processing of complex parts such as turbine blades or medical implants. In aerospace manufacturing, laser interferometers are used for initial calibration to ensure system accuracy, while ball-bar and R-test methods support rapid diagnosis and adjustment during operation. The integrated use of both methods not only enhances system robustness and measurement coverage, but also improves the credibility and generalization capability of error modeling.

Moreover, with the advancement of multi-axis intelligent manufacturing, integrated strategies have become a mainstream trend. Direct measurement provides a foundational basis for validating model parameters used in indirect algorithms, while indirect measurement enhances the system's responsiveness and adaptability to operational errors through dynamic data acquisition. This integrated approach significantly improves the long-term stability, accuracy consistency, and intelligence level of high-end CNC systems, laying a solid foundation for subsequent error modeling, compensation, and closed-loop control.

3.2. Intelligent system

As the demand for accuracy, efficiency, and response speed in complex part manufacturing continues to grow, traditional error identification methods show clear limitations under conditions involving multi-source coupling, dynamic variation, and high-frequency disturbances. Intelligent system technologies offer a new architecture for machine tool error identification and evaluation. By integrating multi-sensor perception, intelligent data analysis, and adaptive control, they significantly enhance perception capabilities and response accuracy in dynamic environments.

3.2.1. Architecture and key technologies. In high-speed multi-axis machining scenarios such as aerospace, automotive, and semiconductor manufacturing, machine tools face sub-millisecond-level dynamic response requirements. The presence of thermo-mechanical coupling, nonlinear servo lag, and multi-source disturbances leads to a highly complex error evolution process that cannot be accurately identified using a single method. At the same time, massive amounts of high-frequency heterogeneous data pose significant challenges to real-time computation and feedback, highlighting the urgent need for high-bandwidth, multi-source fusion, and adaptive error identification technologies.

In recent years, intelligent systems that integrate multi-sensor perception[191,192], AI and machine learning[193,194], digital twin modeling[195,196], computer vision and non-contact measurement[197], big data analysis, and Internet of Things (IoT) technology[198] have become a major trend in error analysis and control. These systems not only realize real-time error monitoring, non-contact recognition, and online modeling, but also significantly improve the precision and speed of prediction and compensation.

Among them, digital twin technology reconstructs machine tool operating states through virtual–real mapping, making it especially suitable for early-stage error modeling, dynamic prediction during operation, and system stability analysis[199–201]. By leveraging real-time data-driven mechanisms and virtual–real fusion, the system can continuously perceive state evolution trends, support dynamic modeling, bias intervention, and closed-loop control—substantially reducing modeling costs and improving response robustness[202–205].

At the same time, machine vision combined with deep learning demonstrates significant advantages in error feature recognition. By using image processing and neural network models, key error features can be accurately extracted under diverse working conditions[206,207]. For example, in addressing thermal error issues, displacement sensors and fiber Bragg grating (FBG) sensors are deployed to monitor thermal deformation fields[208–210], and visual systems are employed for dynamic modeling, enabling thermal error pattern recognition and real-time compensation optimization[211,212].

Figure 9 illustrates the technical pathway and functional modules of the intelligent error identification system, encompassing multi-source data acquisition, feature extraction, intelligent algorithms, error tracing and decoupling, and real-time feedback control. The system is capable of integrated architecture, cross-scale perception, and full-process optimization.

In practical applications, intelligent error identification systems typically include three key stages: First, the error identification stage integrates physical modeling with data-driven methods, combining higher-order statistics and deep learning to achieve accurate extraction and nonlinear modeling of error features. Second, the error tracing and decoupling stage identifies error sources and their transmission paths based on multi-sensor data and simulation models, revealing the multi-source coupling mechanisms. Third, the prediction and control stage builds closed-loop control through real-time data feedback and adaptive strategies, achieving error suppression and system steady-state maintenance in dynamic environments. This architecture provides efficient support for error management in complex manufacturing scenarios.

3.2.2. Application strategies. Based on a digital twin and an AI-integrated intelligent system for error identification and evaluation, this system incorporates four sub-modules: geometric model, physical model, behavioral model, and rule model. These modules are linked via hybrid modeling, enabling dynamic characterization and modeling of multi-physics field coupling errors as well as compensation and control of multi-source errors, as shown in Figure 10.

Figure 10(a) illustrates the functions and components of the four types of sub-models. The geometric model describes nominal deviations in the machine structure, providing a framework for geometric error modeling. The physical model simulates thermal conduction and mechanical deformation, modeling the thermo-mechanical-force coupling response to support the modeling and compensation of thermal and vibration errors. The behavioral model simulates the dynamic response of the machine tool, providing real-time adaptive learning to assist in dynamic error prediction and analysis. The

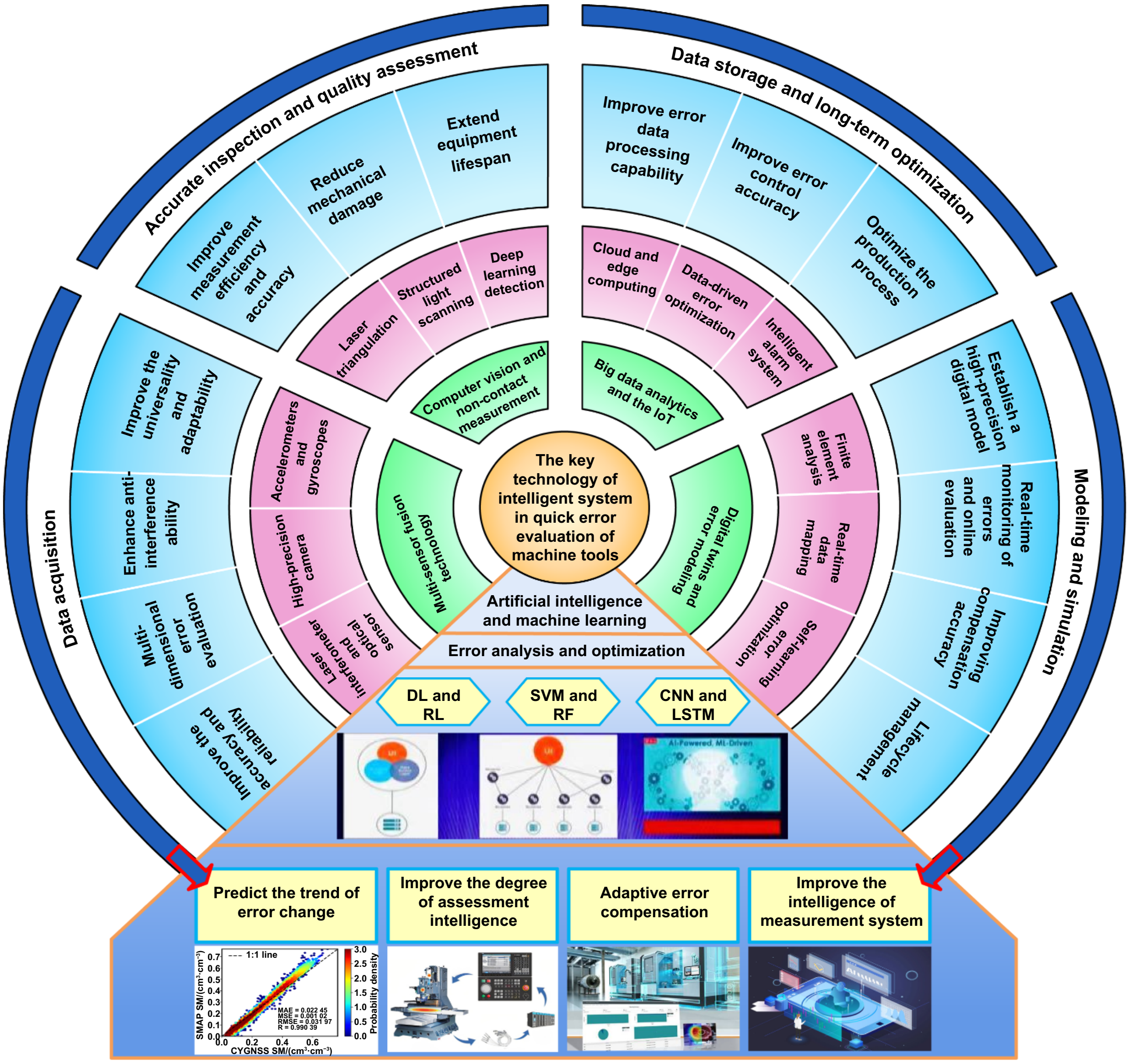

Figure 9. Key technologies and functions of the intelligent system in the evaluation of multi-source errors of machine tools.

rule model offers strategy support for error compensation and control, enabling real-time precision adjustments.

Figure 10(b) presents a complete data processing framework for machine tool multi-source errors, covering error data acquisition and preprocessing, storage and analysis, fusion techniques, and visualization. During machine tool operation, a vast amount of heterogeneous data is generated from the machine entity, digital twin model, and decision control system. The data spans structured, semi-structured, and unstructured formats, with frequencies ranging from 10 Hz to 10 kHz and data volumes from kilobytes to gigabytes, exhibiting high frequency, high heterogeneity, and high dimensionality[216]. To address these characteristics, techniques such as Z-score normalization and principal component analysis (PCA) are used for standardization and dimensionality reduction[217], improving data consistency and model input quality. Data quality is further improved through data cleaning and outlier handling[218,219]. Deep learning, neural networks, and deep reinforcement learning are used for adaptive training and online updating of error prediction models. Training sets are expanded via virtual sample generation[220] and Generative Adversarial Networks (GANs)[221], enhancing model generalization and adaptability. Sequence analysis and error decision fusion techniques provide multi-dimensional support for dynamic precision control[222].

Figure 10(c) outlines five major research focus areas in machine tool error analysis. First, error identification and optimization, where machine vision and sensor technologies improve identification precision and environmental adaptability, providing reliable data for compensation. Second, state awareness and data fusion technologies enable accurate error prediction and control under different working conditions. Third, data-driven error modeling methods, combined with deep learning, improve the recognition of complex error patterns and enhance model realism and applicability. Fourth, error tracking and decoupling techniques reveal the transmission paths of complex error sources, facilitating precise system adjustment. Fifth, real-time compensation and interactive

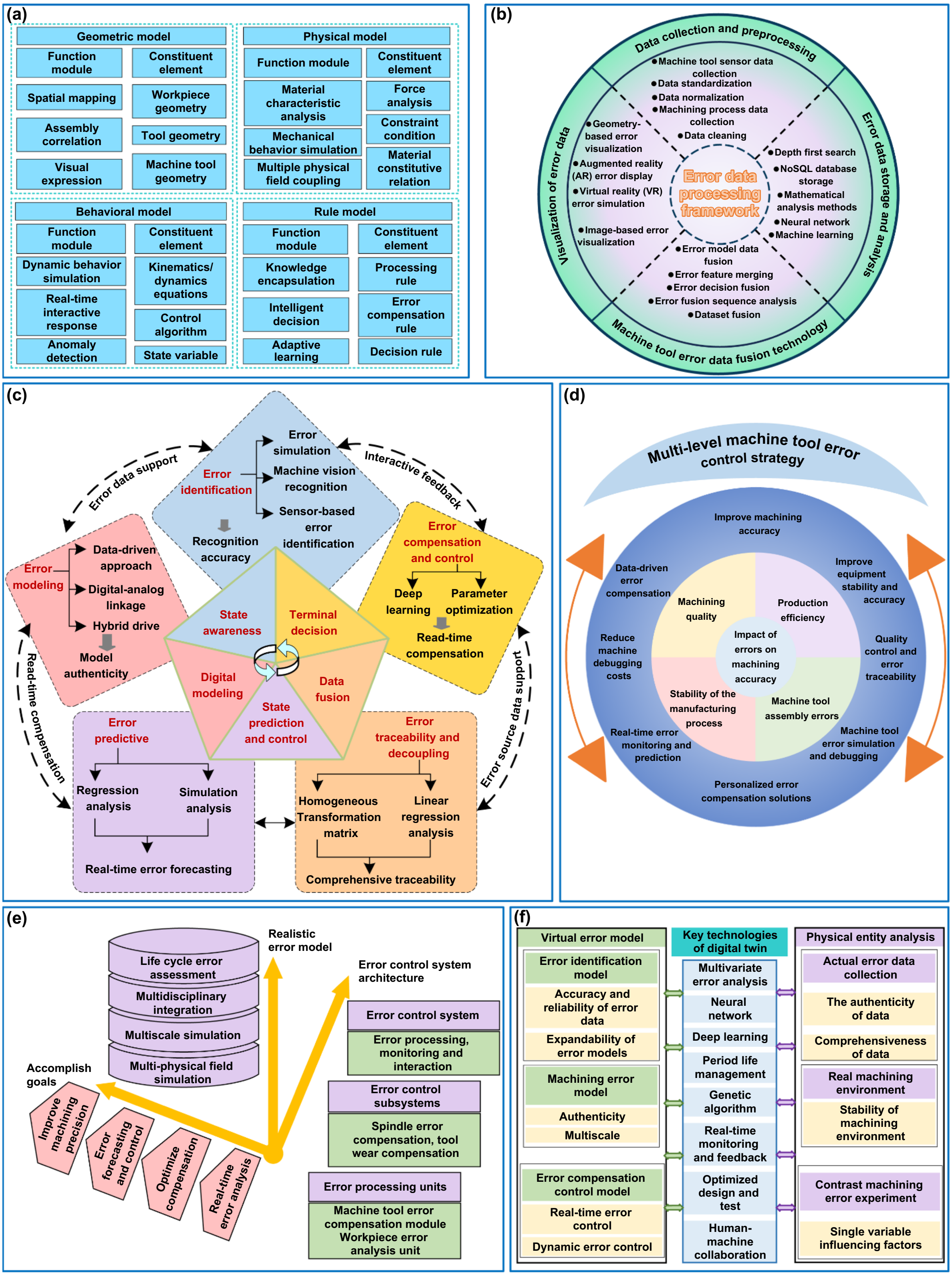

Figure 10. Framework for modeling, analysis and control of multi-source errors in machine tools[213–215]. (a) Functions and composition of sub-modules in digital twin modeling. (b) Machine tool error data processing framework. (c) Key research areas in machine tool error analysis. (d) Machine tool error control and impact on accuracy. (e) Integrated technologies for error modeling, prediction, and compensation. (f) Virtual and real error models in CNC machining.

feedback mechanisms enable immediate error adjustment in dynamic environments, effectively reducing error accumulation. These key areas complement one another and collectively enhance system error control capabilities.

Figure 10(d) summarizes the multi-level machine tool error control strategies and their impact on machining accuracy. From the outermost to innermost levels, it clearly illustrates different technical requirements and optimization goals. The outer layer focuses on improving overall machining accuracy, enhancing equipment stability, and reducing debugging costs, emphasizing the importance of data-driven error compensation and real-time feedback mechanisms. The middle layer addresses machining quality, production efficiency, and the impact of assembly errors on the manufacturing process, reinforcing process stability. The innermost layer delves into the microscopic machining level, exploring the direct impact of errors on precision and promoting micro-level optimization. Through a progressive strategy system combining real-time monitoring, error simulation, and compensation, overall machining accuracy and productivity are systematically improved.

Figure 10(e) summarizes the integrated technical system for real-world machine tool error modeling, prediction, and compensation. It includes multidisciplinary and multi-physics integrated simulation, subsystem modular design, real-time monitoring, predictive analysis, and closed-loop control, forming a systematic, all-encompassing error control architecture. This integration not only enhances error control capability but also, through intelligent and modular design, improves system flexibility, adaptability, and intelligence—driving the advancement of high-precision machining and intelligent manufacturing.

Figure 10(f) summarizes a system framework in CNC machining error research encompassing virtual and real error models, control strategies, and advanced technologies. It presents the latest developments across the full process from error identification, modeling, and analysis to prediction and compensation control. Especially, the high-efficiency coordination between virtual models and physical entities enables high-precision error perception, prediction, and real-time compensation even in dynamic environments, advancing CNC machine tools toward high performance and intelligence.

Despite the advantages of intelligent error identification systems in real-time performance and adaptability, several challenges remain: they heavily rely on large volumes of high-quality data, and insufficient or biased data may lead to model overfitting and performance instability; cross-platform migration is difficult, requiring retraining under different equipment or conditions; deep models lack physical interpretability, making industrial validation difficult; and system deployment relies on multi-sensor configurations and high computational platforms, increasing integration and maintenance costs.

To address these issues, multi-sensor fusion technologies have been widely adopted. By integrating displacement, thermal, force, vibration, and servo feedback signals across multiple domains, these technologies enhance error separation capabilities and system robustness. Fusion algorithms such as Kalman filtering, Bayesian inference, and deep residual networks further improve adaptive modeling and online compensation[223–226]. Moreover, emerging edge computing-assisted fusion mechanisms and high-speed digital twin feedback architectures have shown preliminary success in reducing response latency and improving model update efficiency[227–229].

However, achieving truly high-frequency, scalable, physically interpretable, and cross-platform generalizable intelligent recognition systems remains a key research frontier in intelligent manufacturing[230]. Building hybrid modeling systems that integrate physical mechanisms with data-driven approaches to balance interpretability, generalization, and real-time performance, is a crucial direction for future development—and one of the core goals of the multi-level integrated architecture proposed in this paper.

4. Theoretical modeling

Error modeling aims to accurately describe the internal mechanisms of error generation and propagation. A hybrid modeling approach that combines physical models with data-driven models is used to capture complex multi-physics coupling and nonlinear behaviors, thereby improving model accuracy and adaptability to changing conditions.

4.1. Geometric error modeling

With the evolution of CNC technology and the growing demand for high-precision machining, geometric error modeling has become fundamental to optimizing machining performance and maintaining operational stability[231]. Based on measurement results, geometric error characteristics of translational and rotational axes can be systematically identified, and corresponding mathematical models established to support error propagation analysis, online monitoring, and real-time compensation[232].

These models are applied not only for static structural error analysis but also for predicting and correcting errors under variable conditions, such as thermal deformation, thus maintaining long-term dimensional accuracy[233]. Compensation and calibration techniques based on geometric error models are widely used in industry, effectively improving part dimensional accuracy and tolerance consistency. Two representative methods are homogeneous transformation matrices and screw theory, which are discussed below.

4.1.1. Homogeneous transformation matrix. The homogeneous transformation matrix provides a unified way to represent translation and rotation in space[234]. By combining transformations via matrix multiplication, complex kinematic sequences are simplified—particularly useful in multi-axis CNC kinematic analysis to ensure the end-effector reaches its target position and orientation accurately[232].

The advantage of this method lies in its ability to decompose transformation operations into basic rotation and translation matrices, clearly revealing the impact of each motion component on overall geometric accuracy, thus providing a basis for error traceability and compensation. In addition, the homogeneous transformation matrix method demonstrates good adaptability and scalability in the field of machine tool geometric error modeling. Homogeneous transformation matrices are adept at describing static geometric deviations, but they lack adaptability in dynamic conditions and complex multi-axis interaction scenarios.

4.1.2. Spiral theory. Screw theory, grounded in rigid-body motion principles and introducing concepts like screw axis and pitch, describes simultaneous translations and rotations[235]. Compared with traditional methods, screw theory provides a more intuitive analysis of error coupling in high-speed, high-precision scenarios and is particularly suitable for analyzing five-axis CNC paths[236].

In dynamic machining environments, screw theory leverages instantaneous motion properties to support real-time error modeling and compensation. Its strong differential kinematics capabilities help dynamically optimize tool paths based on trajectory deviations, significantly improving process accuracy and stability[235].

Overall, homogeneous transformation matrices and screw theory each possess unique advantages in sequential transformation processing and complex coupled motion modeling. Their combined use allows systematic identification, quantification, and compensation of geometric errors in machine tools, enhancing machining quality and equipment reliability in high-end manufacturing[237].

4.2. Thermal error modeling

Thermal error modeling plays a key role in achieving high-precision manufacturing and effective thermal compensation control. It integrates information from temperature fields, structural deformation, and end-effector pose deviation to build efficient, interpretable models capable of real-time prediction. With advances in multi-physics simulation, sensor networks, and AI, thermal modeling is evolving toward integration, intelligence, and high precision.

Based on modeling principles, thermal error methods fall into empirical modeling and theoretical modeling[238]. Empirical models treat thermal error as a black box and rely on experimental data to map temperature changes to errors. Theoretical models incorporate heat-transfer theory and structural thermal deformation mechanics to construct physically meaningful mathematical models.

Empirical methods focus on data-driven modeling, using techniques such as least squares[239,240], multiple regression analysis[241], support vector machines[242], artificial neural networks, and fuzzy inference systems[240]. These methods are efficient in complex environments, providing strong adaptation for real-time thermal predictions. Especially, hybrid models like wavelet or dynamic fuzzy neural networks have been widely applied to model spindle thermal drift and feed system thermal error[243,244].

Regression analysis enables quick temperature-to-error predictions without extensive physical modifications, though it struggles with complex nonlinear relations[245]. Machine learning methods address this limitation by capturing intricate temperature-deformation relationships. For example, fuzzy-neural network models predict spindle thermal deformation using temperature data[246]. Traditional sensor fusion methods rely on limited measurement dimensions and encounter difficulties in dealing with constantly changing process disturbances and heterogeneous data fusion, making them unable to cope with non-stationary and complex working conditions.

However, empirical models still face challenges in industrial applications: they are highly dependent on the quality and distribution of samples, lack generalization capability across different machine tools and processes, and suffer from insufficient physical interpretability, making them difficult to support thermal design optimization for machine tools. To address these issues, current research is focusing on physics-informed machine learning, transfer learning, edge computing, and digital twin technologies in an effort to improve model prediction accuracy and adaptability under dynamic thermal fields. For example, a recently proposed spatiotemporal interactive ensemble network integrates time-memory mechanisms with multi-scale fusion strategies, achieving a thermal error prediction accuracy of 97.52% and reducing positioning errors by 90% under small-sample conditions, offering a new approach for digital twin-driven thermal compensation[247].

Theoretical models, based on heat source distribution, structural response, and conduction, are important for machine design optimization. Examples include homogeneous transformation-based thermal error models[248], multi-region temperature/heating models for screw thermal distribution[249], closed-loop iterative models for ball-screw thermal error prediction[250], and spindle thermal drift models stable under speed and cooling perturbations[251]. These methods emphasize thermal source mechanisms and error transfer paths and work best under well-defined structural and boundary conditions.

Finite element analysis (FEA), as an important tool for theoretical modeling, can accurately simulate the heat conduction, convection, and radiation processes in complex structures, and is widely applied in the thermal characteristic prediction of key components such as spindles, lead screws, and bed frames[252]. It supports thermal deformation prediction under varying loads and environments[253–255]. The combination of power-matched temperature control strategies can analytically link temperature and deformation fields to enhance precision stability[256]. However, its large computational load, poor real-time performance, and highly dynamic thermal boundary conditions during the processing process limit its online application capability. At the same time, the model's accuracy is highly dependent on the completeness of geometric, material, and assembly parameters, and is easily affected by structural aging and wear, resulting in insufficient long-term application stability[72,257].

In summary, empirical modeling offers fast deployment and real-time compensation, theoretical modeling supports understanding of thermal mechanisms and system optimization, and FEA excels in complex-structure scenarios. Future directions lie in hybrid modeling that integrates physics and data-driven approaches for interpretable, high-precision, self-adaptive thermal error systems.

4.3. Dynamic error modeling

Dynamic error modeling is critical in precision engineering, particularly for optimizing CNC machine tool performance. These errors manifest as time-dependent changes in tool position, orientation, and process parameters, directly affecting part quality. To analyze and predict machine tool dynamic behavior, modal analysis and transfer-function modeling are widely used.

Modal analysis is a key technique for studying the dynamic behavior of mechanical structures. It is used to identify the inherent vibration modes of machine tools and provides dynamic characteristics such as natural frequencies, mode shapes, and damping ratios[258]. These parameters are essential for predicting the response of a machine tool under dynamic loads and help identify the primary vibration modes that may lead to machining errors. By evaluating these vibration modes, engineers can design optimization strategies—such as modifying the structural design or introducing damping measures—to improve the operational stability and machining accuracy of the machine tool.

During the modal analysis process, various methods can be used to obtain the necessary data. Experimental modal analysis typically involves applying a known input (such as an impact hammer or shaker) to excite the machine tool and using accelerometers or other sensors to measure the response[259]. From these measurements, the Frequency Response Function (FRF) can be derived, which allows for the calculation of the system's modal parameters. In addition, mathematical modeling and numerical simulation methods can be employed to predict the modal characteristics of machine tool components under different working conditions, enabling the optimization of design and operational strategies.

Transfer function modeling, on the other hand, is based on the mathematical relationship between system inputs and outputs, using the Laplace transform to describe dynamic response characteristics[260]. Through system identification methods and experimental data collected during operation, a dynamic behavior model of the machine tool can be established. Transfer functions not only facilitate simulation and analysis prior to physical adjustments, but also serve as an important tool for error compensation and dynamic optimization.

Modal analysis and transfer function modeling are complementary approaches in dynamic error modeling. The former focuses on revealing the internal dynamic characteristics of the machine tool structure, while the latter emphasizes the input-output response relationship. Together, they support the development of machining error prediction and compensation strategies[261]. Based on these methods, researchers can systematically enhance the dynamic stability and machining accuracy of machine tools through structural design optimization, control system integration, real-time monitoring, and adaptive adjustment.

In summary, systematic modeling of dynamic errors using modal analysis and transfer function modeling not only provides a theoretical foundation for reducing machining errors and improving product consistency but also lays the groundwork for future efficient and intelligent dynamic error control technologies. With the continuous advancement of manufacturing and intelligent technologies, this field holds vast development potential.

4.4. Integrated error modeling

In high-end manufacturing, traditional methods mostly rely on offline modeling and fixed mathematical structures, making it difficult to cope with the dynamic evolution of errors under complex working conditions. These methods lack real-time performance and adaptability, and the error modeling processes are independent of each other, lacking the capability for collaborative analysis and fusion of multi-source errors, thus hindering coordinated compensation. The cross-coupling among geometric, thermal, cutting force, dynamic, and control errors has become a critical bottleneck in maintaining machine tool accuracy[262,263].

DT technology introduces a cyber-physical synchronization mechanism, enabling real-time state monitoring, multi-physics interactive simulation, and dynamic adaptive modeling, thereby reshaping the paradigm of error management. Especially when combined with AI methods, such as Long Short-Term Memory (LSTM) networks and other advanced machine learning techniques, error modeling has shifted from a static, offline approach to a dynamic, full-process accuracy management framework. This systematically addresses core issues of traditional modeling in nonlinear behavior description, uncertainty handling, error coupling characterization, and real-time adaptability. Studies have shown that virtual machine tool technologies[264] have already achieved integrated modeling of spindles[265], feed systems[266], and cutting processes[267], effectively supporting machining performance prediction and operational optimization[268,269]. Integrated error modeling unifies modeling and multi-source data fusion, revealing the interaction and transmission mechanisms among errors and providing systematic support for precision control.

This section summarizes an integrated error modeling framework based on the "perception–modeling–control" closed-loop concept, integrating multi-physics modeling, data-driven, and hybrid modeling approaches to support dynamic error prediction and lifecycle-wide compensation. The subsequent content elaborates on four aspects: core sub-models, multi-level modeling strategies, real-time control, and evaluation systems.

4.4.1. Core sub-model.

In the integrated error modeling framework, four core sub-models are defined: geometric model, physical model, behavioral model, and rule model. These describe the generation and evolution mechanisms of errors from structural, mechanistic, response, and strategic levels, respectively. Figure 11 illustrates key enabling technologies and representative application platforms for each core model.

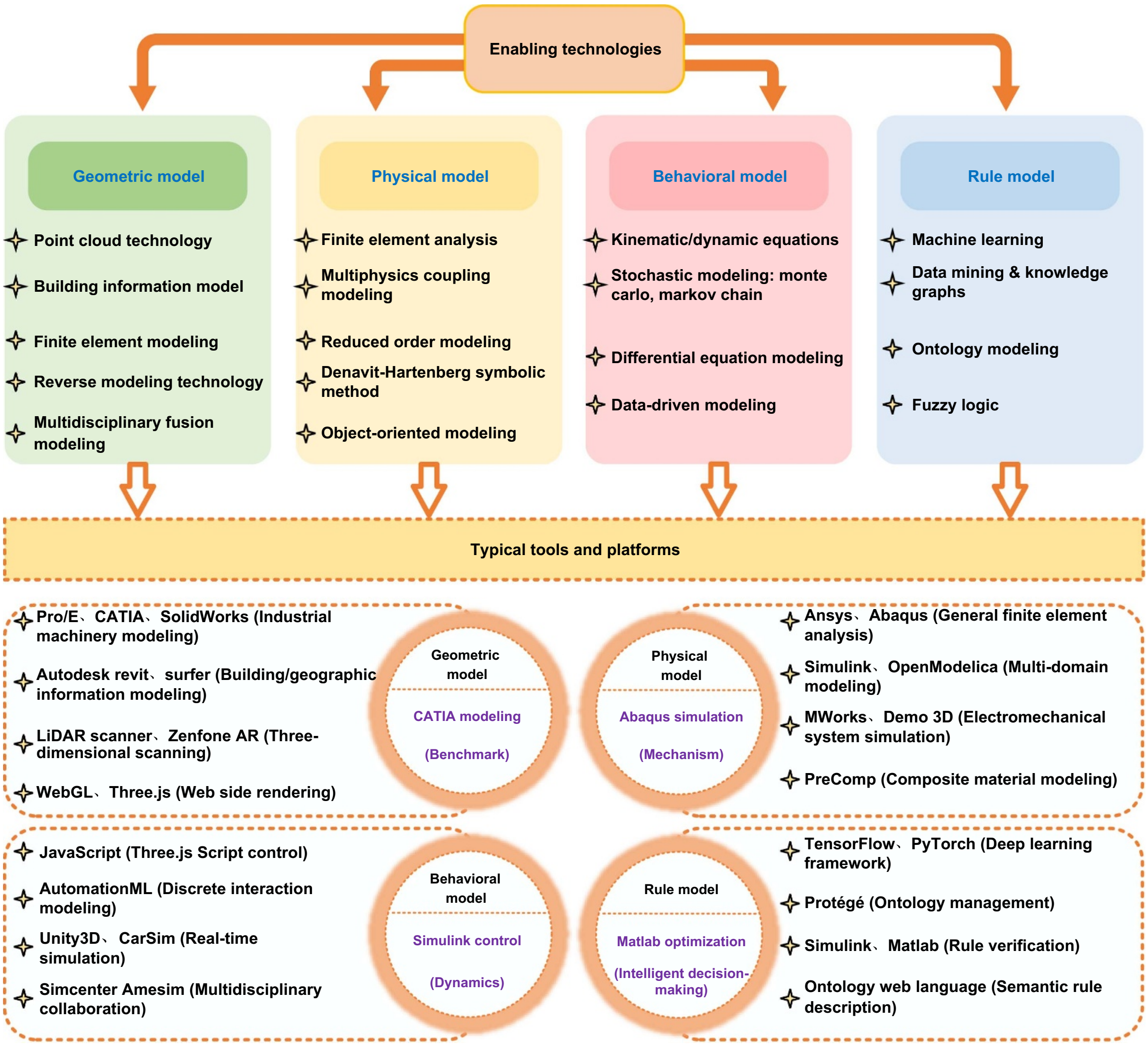

Figure 11. Enabling techniques, typical tools and platforms of the core sub-model[270].

The geometric model provides the spatial reference for error modeling, focusing on the geometric structure and assembly relationships of machine tool components. Through virtual assembly and interference analysis, it can optimize structural layout, shorten error transmission paths, and enhance system stability and machining accuracy.

The physical model, based on thermo-mechanical-structural multi-physics coupling, describes the mechanisms of error sources such as thermal deformation, cutting forces, bearing wear, and backlash, and quantifies their impact on terminal accuracy. It serves as the theoretical foundation for error traceability and control.

The behavioral model characterizes the dynamic response and error variation of machine tools during operation. By combining multibody system dynamics simulation and trajectory tracking techniques, it enables error trend prediction and constructs sensor-driven adaptive control mechanisms to realize dynamic compensation and real-time optimization during machining.

The rule-based model integrates the outputs of the above models to generate precision control strategies. Relying on deep learning (e.g., LSTM) and knowledge graphs, it combines process knowledge with data-driven methods to optimize compensation schemes online and adaptively adjust process parameters based on real-time feedback, thereby enhancing system robustness.

4.4.2. The methodology of modeling. Error modeling driven by digital twin technology primarily relies on three modeling approaches: physics-based modeling, data-driven modeling, and hybrid modeling. Each corresponds to different integration strategies for sub-models, forming a multi-paradigm collaborative mechanism aimed at achieving high accuracy, strong real-time performance, and intelligent evolution capabilities in error modeling. The effective operation of this modeling system depends on two core capabilities: first, real-time updating ability, which enables dynamic response to changes in machining states; second, a model consistency

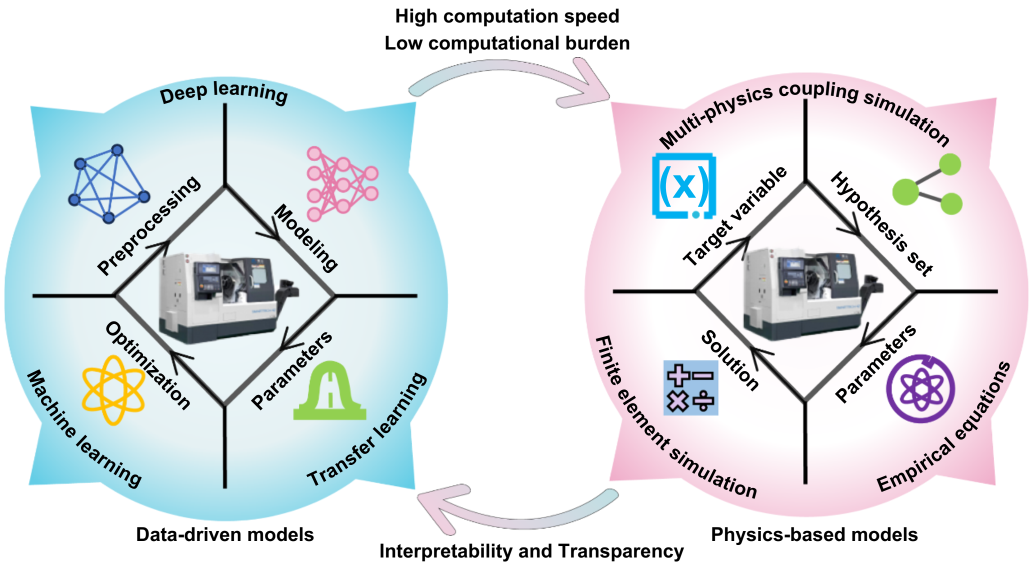

Figure 12. Complementary relationship between physics-based and data-driven models. Reprinted from[284], Copyright (2022), with permission from Elsevier.

maintenance mechanism, which ensures predictive accuracy and system stability throughout the modeling cycle[271].

Physics-driven modeling focuses on constructing high-fidelity models based on physical laws, commonly employing theoretical methods such as multibody dynamics and finite element analysis. It primarily aims to quantitatively describe the coupled processes of geometry, thermal, and mechanical fields. Representative studies include: refined modeling and experimental validation of geometric errors in five-axis machine tools[43]; thermo-mechanical coupling models revealing the evolution of composite errors during the cutting process[272]; thermal-mechanical coupling calculation methods for thermal deformation errors in motorized spindles[273]; and dynamic contact modeling of fixtures/spindles to enhance adaptability to complex machining environments[274].

High-fidelity digital twin models constructed using these methods can accurately model 41 geometric errors of machine tools (including 21 translational errors and 20 angular errors). For instance, the positioning error of the X-axis at -200 mm is measured at 0.003 0 mm, while angular errors are precisely fitted using third-order polynomial functions. By comparing the predicted toolpath with the actual trajectory measured by a coordinate measuring machine (accuracy: 1.5 µm + L/350), the model's high fidelity and usability in toolpath contour error prediction are verified[43].

Although physics-driven modeling offers excellent physical interpretability and prediction accuracy, it still faces challenges such as high computational cost, long modeling cycles, and insufficient real-time performance under complex and high-speed machining conditions, which limit its widespread application in online control.

Data-driven modeling uses machine learning methods (e.g., Support Vector Machine (SVM), Convolutional Neural Network (CNN), LSTM) to mine nonlinear mapping relationships from sensor data and establish efficient error prediction models for rapid forecasting and online compensation. Representative applications include spindle thermal drift prediction[275], tool wear modeling[276], and bearing dynamic characteristics analysis[277]. Edge computing and multi-sensor fusion have significantly improved sensing efficiency[278,279]. However, the "black-box" nature and dependence on high-quality data limit generalization and autonomous decision-making capabilities[280,281].

Hybrid modeling combines the strengths of physics-driven and data-driven methods. By retaining the physical model's interpretability while introducing dynamic corrections via data-driven techniques, it achieves both high precision and strong robustness in error modeling. As shown in Figure 12, this modeling strategy has been widely applied in typical scenarios such as spindle thermal error, tool path error, and contour error modeling[203,282,283].

With the development of 5G communications and simulation-integrated CNC cores, researchers have achieved high-frequency synchronization between physical entities and virtual models[285,286], extending hybrid modeling to real-time visualization of tool paths and cutting forces[287,288], condition monitoring of grinding machines[289], tool failure prediction[290], real-time monitoring of spindle dynamic properties[291], and online prediction of workpiece surface roughness[292], thereby enhancing intelligent maintenance capabilities.

In recent years, hybrid modeling has shown strong potential in performance degradation prediction. By combining contour modeling of machining characteristics, thermal deformation, and dynamic cutting force modeling, wear mechanism modeling, and particle filter-based dynamic updates, researchers have achieved high-precision prediction of machine tool thermal error, feed transmission error, contour error, and spindle performance degradation[47,271,293–297]. This strategy not only optimizes overall machine performance but also improves the adaptability of process parameter to changing conditions. By introducing adaptive triggering mechanisms, a

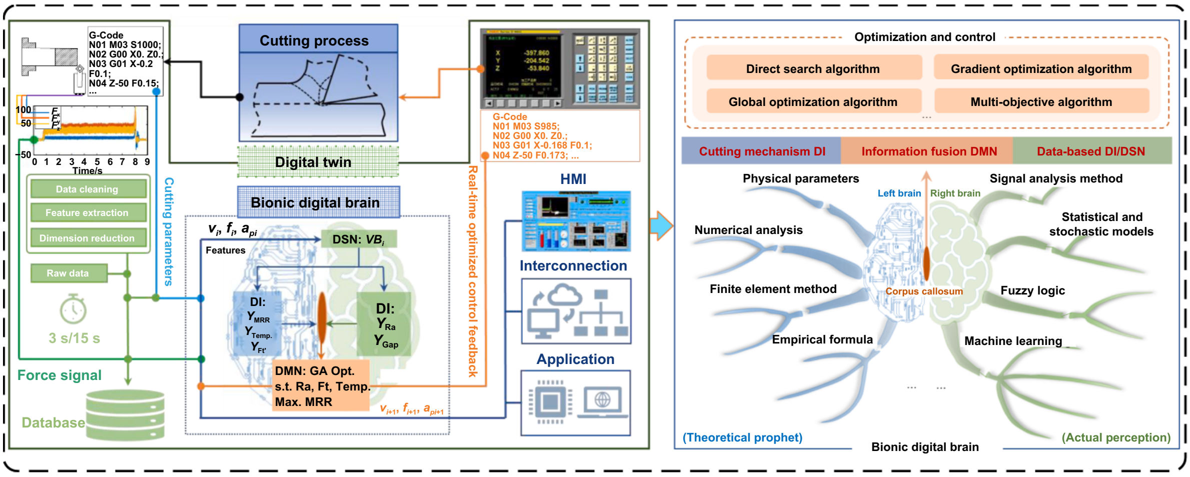

Figure 13. Bionic digital brain for realizing the digital double-knife cutting process. Reprinted from[309], Copyright (2023), with permission from Elsevier.

low-cost, efficient multi-source feature dataset updating system has been built, significantly enhancing model dynamism and practicality[34].

In terms of multi-source feature modeling, researchers have constructed multi-level electromechanical-hydraulic digital-twin models covering mechanical, hydraulic, control, and electrical subsystems, enabling virtual debugging and dynamic updating of servo parameters[298]. Furthermore, the proposed digital twin–driven composite control strategy has improved the overall control accuracy of feed systems[299]. When addressing multi-axis coordination errors in high-speed CNC machining, researchers developed an electromechanical-coupling model of multi-axis feed systems based on joint stiffness and friction disturbances. Through unbiased least squares identification of disturbance parameters and joint stiffness, and trajectory fitting using Ferguson curves, they achieved efficient estimation and dynamic pre-compensation of contour errors, significantly improving the accuracy and stability of five-axis machining[300]. Additionally, by combining mechanism-based and data-driven models, researchers realized dynamic optimization of cutting parameters and online evaluation of machining stability[301].

For quantitative analysis of high-fidelity hybrid models, particularly under conditions of limited samples and distribution differences in spindle thermal error modeling, a digital twin strategy combining physical simulation modeling and deep transfer learning has been proposed. By building a high-fidelity model of spindle thermal behavior to generate synthetic data and using a distance-guided domain-adversarial network for source-to-target knowledge transfer, this method achieves high-precision modeling even with minimal real-world samples, boosting fitting accuracy by 11.73%[40].

Simultaneously, to improve system responsiveness, researchers have explored instruction-signal excitation, integration of CAM and simulation, and edge–cloud deployment strategies[302–305]. In modeling complex structures such as gears and thin-walled workpieces, techniques including Gaussian process regression and modal modeling have been used to build dynamic error prediction and stability analysis frameworks[269,306].

On the industrial application front, multiple cases have demonstrated the practicality and feasibility of physics–data hybrid modeling. For example, in aerospace aluminum alloy milling, researchers developed a milling force prediction model based on spindle speed, tooth feed rate, and milling width. The neural network's maximum prediction error was only 4.34%, showing strong agreement between simulation and experimental data—a practical, low-cost solution[307]. In spindle thermal error modeling of CNC machine tools, a hybrid approach combining physical mechanisms and data-driven methods decomposed the complex spindle system into multi-link structures; by modeling linear deformation components with an equivalent-area method and non-linear components with multi-module LSTM networks, the residual error of thermal prediction was reduced by 45%[308].

Although hybrid modeling holds great promise, practical implementation still faces many challenges, including high-frequency synchronization between heterogeneous sensor data and physical models, control stability issues in multi-time-delay data fusion, high computational overhead during model execution, and controlling structural complexity while maintaining real-time performance. Industrial deployment typically requires deep coordination with hardware acceleration and system architecture, making integration challenging. Current research is gradually overcoming these bottlenecks through multi-rate scheduling, hierarchical modeling, and edge–cloud collaboration mechanisms. Moreover, the Bionic Digital Brain (BDB)—the intelligent core of the Digital Twin Cutter Process (DTCP)—leverages "digital neurons (dn)" to fuse left- and right-brain information and output optimal control schemes in real time, achieving real-time monitoring, prediction, optimization, and control of the cutting process[309], as illustrated in Figure 13.

As shown in Table 3, comparisons of machine tool error modeling and digital–physical synchronization methods

Table 3. Typical machine tool error modeling methods and their main limitations.

Category	Content	Disadvantages	References
Physical-driven model	Geometric and kinematic analysis, thermal error and dynamic modeling	High complexity, uncertain parameters, poor adaptability, difficult model validation	[43, 272, 274]
Data-driven model	Machine learning-based analysis of processing data, extracting key features for modeling	Poor interpretability, complex algorithms, poor generalization	[40, 199]
Hybrid model	Combines physical mechanisms with data-driven adaptive learning modeling	High complexity, difficult to understand and maintain	[34, 200, 313]
Model-data connection	Achieves real-time synchronization and interaction between physical entities and virtual models	Difficult real-time synchronization and interaction	[277, 314]

reveal that although hybrid modeling offers significant potential, key challenges remain around model complexity control, system integration, and efficient synchronization. Especially in high-speed, dynamic environments, breakthroughs are still needed in real-time integration of heterogeneous multi-sensor data, further improving synchronization and generalization capabilities of digital-twin control systems[310–312].

Physics-driven modeling emphasizes the interpretability of governing laws, relying on physical principles and prior knowledge. It mainly integrates geometric and physical models to form high-fidelity error descriptions. Data-driven modeling highlights rapid response capability, focusing on the use of sensor data and machine learning methods, and relies on behavioral and rule-based models to achieve dynamic prediction and compensation. Hybrid modeling strikes an optimal balance between the two by enabling model collaboration, thereby enhancing the accuracy and adaptability of error modeling. Together, these three modeling paradigms constitute the core pathways of digital twin–based error modeling, enabling the concrete construction and optimization of models.

Table 4 summarizes the applicability, advantages, and limitations of machine tool error models under different modeling approaches, providing critical support for a precision assurance system geared toward real-time prediction and intelligent control. Within the overall framework, models and modeling methods exist in a parallel and complementary relationship: sub-models define the content and perspective of error modeling, while modeling methods provide the implementation paths and optimization mechanisms. Their synergistic interaction jointly constructs a machine tool error modeling and control framework oriented toward dynamic evolution and real-time updating, supporting full-lifecycle management of precision retention and performance optimization.

4.4.3. Modeling framework and real-time closed-loop control system. Digital twin technology realizes the digital representation and adaptive control of machine tool errors by constructing geometric, physical, behavioral, and rule models, and combining physics-driven, data-driven, and hybrid modeling methods. Its core lies in mapping physical entities into digital manufacturing modules, acquiring key parameters such as position, temperature, vibration, and cutting force in real time, and integrating them with virtual simulation data such as trajectory, thermal stress, and error information to establish a bidirectional interaction and closed-loop control system between physical and virtual spaces[207]. When integrated with AI technologies, this system gains capabilities in real-time error monitoring, dynamic prediction, and intelligent decision-making, significantly improving machining accuracy and control efficiency under complex and dynamic environments[315–317].

This interaction mechanism relies on two types of key data sources: historical data to improve modeling accuracy, and real-time data to support dynamic response and online compensation[318,319]. Real-time data-driven analysis not only enables error visualization but also provides theoretical support for process parameter optimization[320,321]. Figure 14 presents the multiple types of data involved in the machining and simulation process and their respective modeling paths, offering an intuitive reference for understanding data requirements in the modeling workflow.

To achieve high-precision and real-time error control, this study proposes a multi-level error modeling and management framework that integrates digital twin and AI technologies, consisting of five core components: the physical module, virtual module, data module, connection module, and service module[325]. This framework runs throughout the entire process of physical perception, virtual simulation, and intelligent decision-making, establishing a systematic integration mechanism and closed-loop control capability, as shown in Figure 15.

This integrated framework builds an intelligent error control system suitable for complex manufacturing scenarios through multi-source data acquisition and fusion, hybrid-driven modeling, and real-time feedback control. Specifically, the physical module targets the actual machining environment, monitoring geometric errors, static responses, dynamic behaviors, and motion control characteristics of the machine tool body, key components, workpieces, and manufacturing

Table 4. Comparison and analysis of multi-model technology systems for machine tool error modeling and control.

Model category	Function	Main technologies and methods	Key application areas	Relation to machine tool errors	Advantages	Disadvantages
Geometric model	Spatial reference mapping, structural optimization	Geometric modeling, virtual assembly, interference analysis	Stability improvement, error transfer path optimization	Fundamental for error modeling, directly affects machining accuracy	Simple and easy to understand, suitable for static error analysis; can accurately capture geometric forms	Unable to handle complex dynamic behaviors; weak adaptability to complex error sources
Physical model	Multi-physics coupling error mechanism analysis	Multi-physics simulation, thermal deformation and mechanical modeling	Error source analysis and prediction	Key source of physical errors (thermal deformation, mechanical loads, etc.)	In-depth analysis of error mechanisms; adapts to complex error sources	Large computational load, high experimental verification requirements
Behavioral model	Dynamic response and real-time trajectory tracking	Multi-body dynamics simulation, real-time data collection and dynamic modeling	Dynamic error prediction and real-time compensation	Dynamic errors caused by motion trajectory changes	Real-time dynamic prediction; suitable for motion error analysis	Complex modeling, highly susceptible to external disturbances
Rule-based model	Intelligent decision-making and adaptive compensation	LSTM, deep learning, knowledge graphs	Automatic error compensation and process dynamic optimization	Optimizes error compensation based on intelligent algorithms	Strong adaptability, can handle complex relationships; continuous learning and optimization	Requires a large amount of training data; poor model transparency
Physics-driven modeling	High-fidelity modeling based on physical laws	Multi-body dynamics, finite element analysis, thermal-structure coupling	High-fidelity error twin and complex condition modeling	Supports in-depth analysis of error mechanisms	High-precision modeling, detailed error source traceability	Complex computation, poor real-time performance
Data-driven modeling	Error trend prediction based on data	SVM, DNN, LSTM, CNN, etc.	Thermal error prediction, tool wear modeling, error optimization	Data-driven modeling of error evolution laws	Adapts to complex non-linear systems; continuous model optimization	Requires high-quality data; lacks mechanistic interpretability
Hybrid modeling method	Collaborative optimization of physical and data models	Integration of physical and data models, time-series analysis	Improves error prediction accuracy and adaptability	Combines physical and data models, enhancing prediction and control	Combines the advantages of both model types, improving accuracy and robustness	Complex modeling and optimization, high resource requirements
Digital twin system	Real-time synchronization and status monitoring	Edge computing and cloud computing collaboration, bidirectional data flow	Real-time monitoring, dynamic optimization, accuracy improvement	Supports full lifecycle error management and optimization	Strong real-time monitoring and dynamic adjustment capabilities	High implementation cost, complex technology, high infrastructure requirements

(Continued.)

Table 4. (Continued.)

Model category	Function	Main technologies and methods	Key application areas	Relation to machine tool errors	Advantages	Disadvantages
Multi-source data fusion	Integration and analysis of sensor and process data	Multi-sensor fusion, edge computing, intelligent algorithms	Real-time error factor identification and prediction accuracy improvement	Multi-source information fusion enhances error perception	Enhances system robustness and real-time performance	Difficult data synchronization, susceptible to noise interference
Performance optimization and control	Adaptive control and optimization based on twin systems	Particle swarm optimization, deep learning, online fault prediction	Stability improvement, fault warning, and performance optimization	Optimizes control strategies, reducing error impact	Dynamic adjustment of control strategies, improving machine tool performance	High real-time requirements, complex optimization process

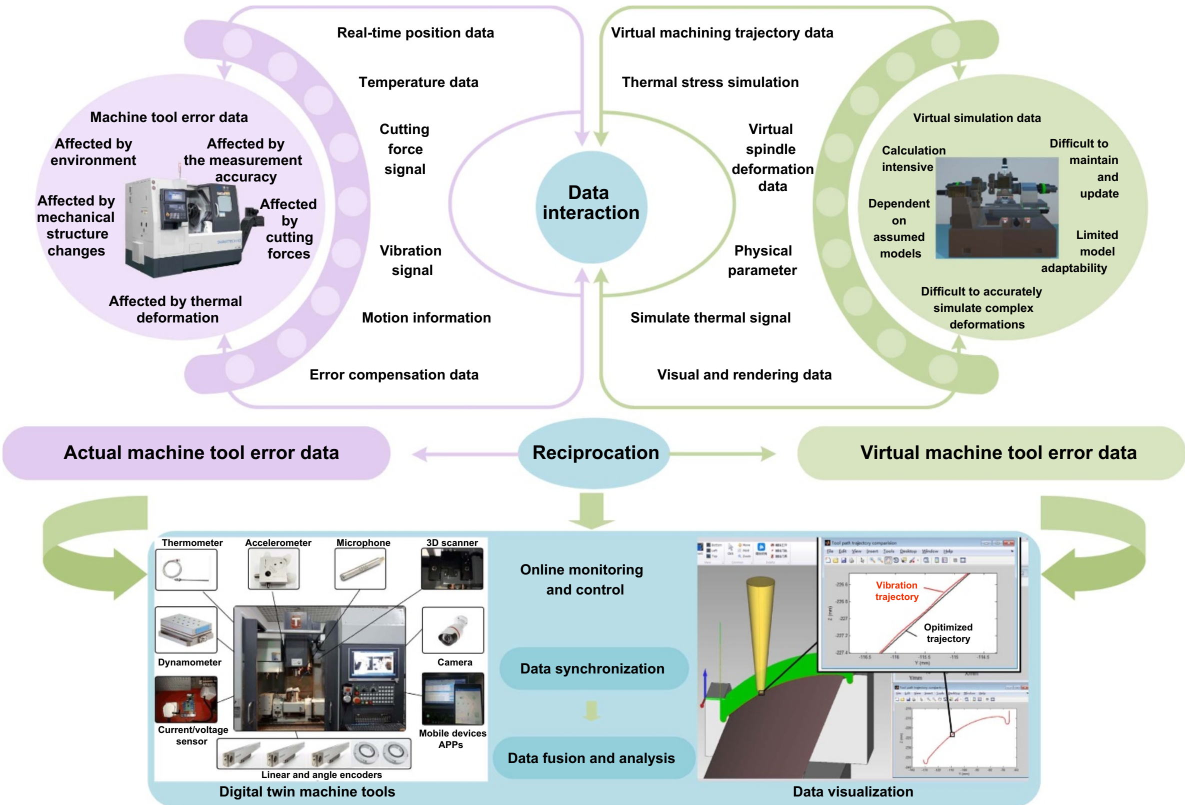

Figure 14. Error modeling technology based on real-time data and virtual simulation-driven approach[288,322–324]. Adapted from[288], with permission from Springer Nature. Reproduced from[322]. CC BY 4.0.

environment[331,332]. With high-resolution sensors and feedback mechanisms, it can rapidly perceive the error evolution process, provide high-quality training data for subsequent modeling, and enhance model accuracy through closed-loop correction mechanisms. The virtual module focuses on high-fidelity simulation, mapping dynamic features in the machining process in real time, such as trajectory changes, thermal stress distribution, and spindle structure deformation. Relying on multi-sensor collaborative measurement, it aligns and cross-verifies virtual and physical measurements to improve the reliability of error prediction and the accuracy of compensation decisions[312]. The data module, as the information hub, leverages IoT, 5G communication, and edge computing platforms to collect and integrate multi-source information in real time, including position deviation, temperature gradients, and servo errors. It provides continuous data support for model training and adaptive optimization[333]. The connection module, as the integration hub of the system, connects heterogeneous models for multi-physics error modeling, serving as the key to capturing nonlinear interactions, cross-domain dependencies, and transient system behaviors under dynamic machining conditions. The service module targets the final application layer. Based on model predictions and data-driven results, it enables real-time visualization of machine tool error

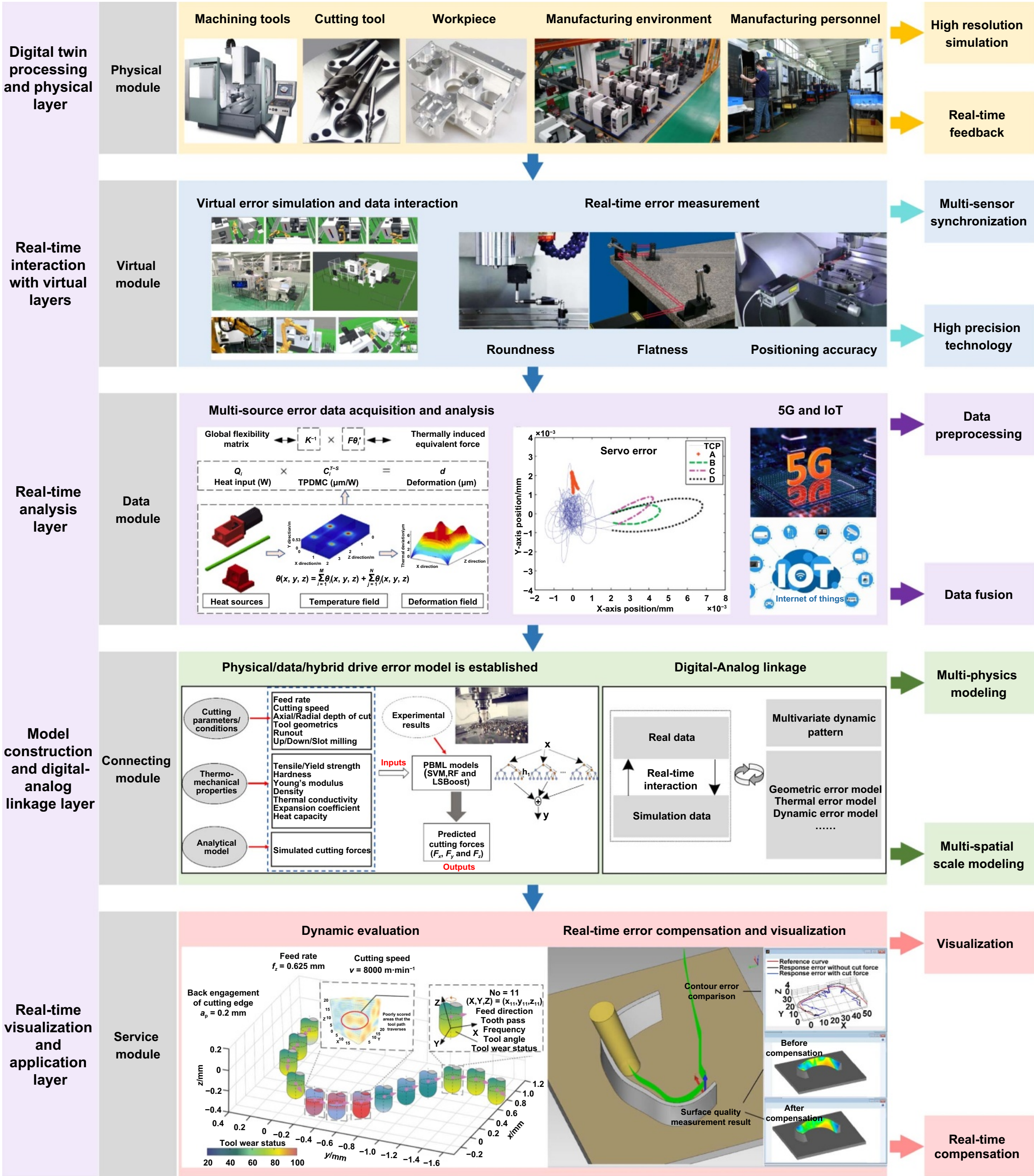

Figure 15. Integrated error modeling framework driven by digital twin[6,117,205,288,326–330]. Adapted from[6], with permission from Springer Nature. Reprinted from[117], Copyright (2021), with permission from Elsevier. Adapted from[288], with permission from Springer Nature. Reproduced from[326]. CC BY 4.0. Reprinted from[329], Copyright (2013), with permission from Elsevier. Reprinted from[330], Copyright (2024), with permission from Elsevier.

states, dynamic prediction, and process parameter optimization, providing intelligent decision support for operators and automated control systems.

The intelligent error control system established by this integrated framework, designed for complex manufacturing scenarios, continues to have broad expansion potential as sensor technology, data processing capabilities, and AI methods evolve. Future research can focus on the following directions: first, introducing adaptive modeling mechanisms to enhance the generalization and robustness of the system under different machine types and changing working conditions; second, optimizing data acquisition and processing to improve the real-time responsiveness of the system, reduce control latency, and enhance dynamic compensation capability; additionally, developing standardized interface protocols to enable efficient data interconnection and collaboration among manufacturing resources, facilitating cross-platform sharing and the upgrade of intelligent manufacturing systems.

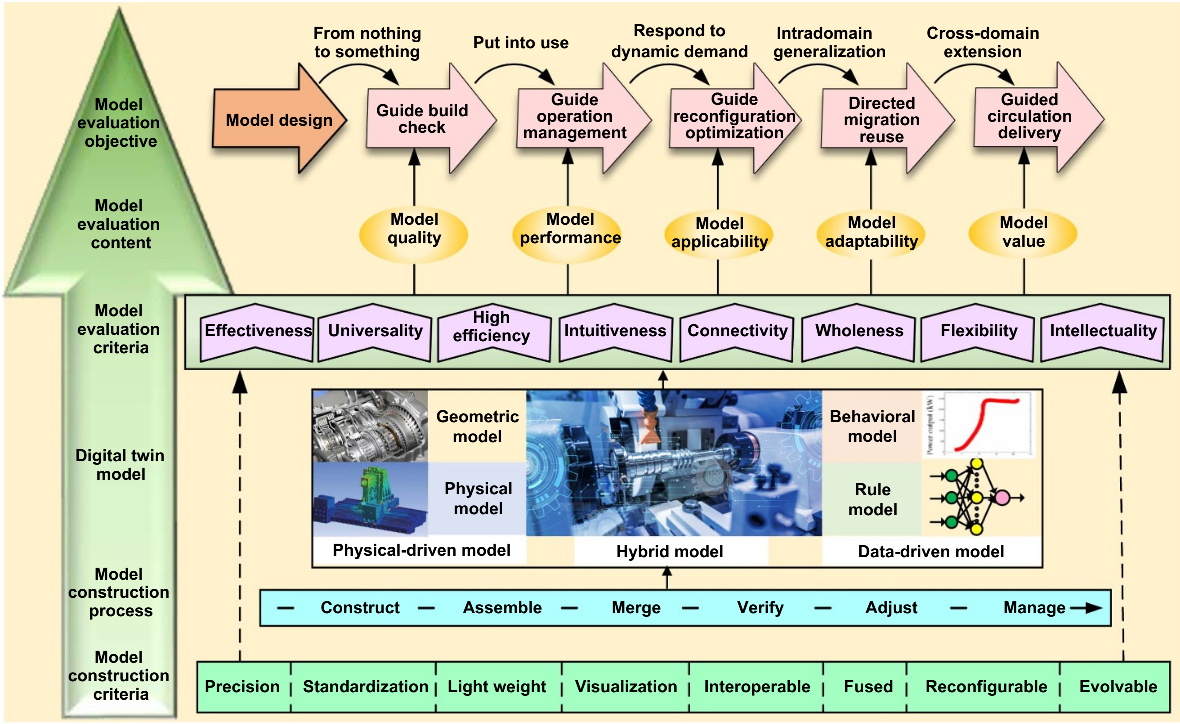

Figure 16. Evaluation system of integrated error modeling[334–338].

4.4.4. Model evaluation system. To standardize the processes of model construction, evaluation, and deployment, Figure 16 proposes a lifecycle-oriented integrated error modeling evaluation index system. This system evaluates model accuracy, real-time performance, stability, and scalability, aligned with application scenario characteristics, providing systematic standards for the construction, evaluation, optimization, and application of integrated error models. It possesses strong engineering guidance value. Its broader application is expected to enhance the coordination and standardization of digital twin system development within industrial enterprises and research institutions, laying a theoretical and methodological foundation for the intelligent transformation of manufacturing.

However, the evaluation system still faces several challenges in practical deployment, such as insufficient standardization of models, high complexity of reconstruction, and the adaptability and compensation effectiveness of intelligent control algorithms under complex conditions, yet to be fully validated. Therefore, future research should focus on: modular optimization of model structures, reinforcement learning mechanisms for control algorithms, and scenario-oriented lightweight deployment strategies. These efforts aim to continuously improve the robustness, adaptability, and sustainable evolution capacity of the system.

5. Traceability and decoupling

In high-precision machine tool machining, error traceability and decoupling are critical steps for identifying error sources and understanding their formation mechanisms. Error traceability aims to establish causal relationships between observed errors and specific machine tool components or subsystems, enabling targeted improvements. Error decoupling is used to separate intertwined error components to prevent mutual amplification and cumulative degradation, allowing various error compensation strategies to be efficiently executed within subsystems without interference. By precisely locating systematic errors, a solid technical foundation is provided for formulating effective compensation and control strategies.

5.1. The importance of traceability and decoupling

During long-term operation, machine tools are subject to the combined influence of wear, temperature fluctuations, vibration, and other factors, inducing complex systemic errors. Error traceability emphasizes precise tracking of error roots, supporting targeted optimization of critical components and process flows[36]. It covers local defects, misalignment issues, and systematic deviations in design[38]. A thorough understanding of error causes aids in implementing targeted repair and preventive maintenance, improving equipment reliability and process stability.

Decoupling analysis aims to separate highly coupled error sources into independent components that can be identified and modeled separately, in order to enhance the accuracy and efficiency of error modeling and compensation control[339]. Strong coupling commonly exists among machine tool structural deformation, thermal effects, dynamic responses, and cutting forces. If not effectively decoupled, it will significantly undermine the adaptability and precision of error diagnosis and control strategies.

In practice, error traceability and decoupling typically rely on multimodal sensor fusion and intelligent data analysis methods[39]. Modern machine tools integrate sensors for temperature, vibration, force, displacement, etc., to monitor key physical variables in real time. By combining principal component analysis (PCA), wavelet transforms, finite-element thermal–structural coupled modeling, and machine learning methods, error features can be effectively extracted and their causes classified and identified[37]. Effectively implementing error traceability and decoupling significantly enhances machine tool accuracy and product consistency, reduces rework rates and downtime, and achieves dual optimization of manufacturing cost and benefit[340,341]. Error traceability and decoupling form the core support for error control in efficient, intelligent, and sustainable manufacturing processes.

5.2. Traceability method

Machine tool errors result from the coupling of various error sources. To accurately identify error origins and implement effective compensation, sensitivity analysis and error budgeting are two key traceability techniques.

Sensitivity analysis establishes a mathematical mapping between system inputs and error outputs, evaluating the impact of each parameter on machine tool accuracy. It not only helps identify key control variables in the error transmission chain—providing data support for design optimization and compensation strategies—but also quantifies error source contributions, determining the influence of system variables on overall machine performance[342]. It provides a theoretical basis for pre-compensation in the assembly stage and enhances feed-forward control capability in the design phase[343].

Error budgeting systematically classifies and quantitatively evaluates geometric, thermal, and dynamic error sources, setting tolerance boundaries for each error source. This provides decision support for tolerance design and manufacturing error control, balancing cost and performance[109,344].

Sensitivity analysis and error budgeting each offer unique advantages in error identification and control: sensitivity analysis is used to identify dominant variables, while error budgeting provides a perspective for evaluating cumulative errors. The combination of both can systematically guide error identification and compensation throughout the entire process of machine tool design, commissioning, operation, and maintenance, significantly enhancing the relevance and effectiveness of control strategies.

Traditional traceability methods struggle with dynamic coupling behavior during machining processes. In contrast, digital twin technology—through multi-physics modeling and real-time data-driven analysis—offers new solutions for dynamic error traceability and compensation. It has demonstrated advantages in thermal–force coupling, vibration response, and material behavior modeling, especially in complex deformation prediction such as residual stress evolution[345], plastic deformation simulation[346], and fatigue crack propagation tracking[347]. In the future, integrating dynamic traceability mechanisms within digital twin platforms will further enhance the real-time and intelligent levels of error modeling, promoting the development of high-precision intelligent manufacturing.

5.3. Decoupling technology

Machine tool error sources are characterized by strong coupling, multi-scale and multi-physics interactions, making decoupling technologies the core support for achieving precision machining, optimizing control strategies, and enhancing system robustness.

The goal of error decoupling is to separate error sources under different physical mechanisms—geometric error, thermal error, cutting force error, and dynamic error—so they can be modeled, analyzed, and compensated independently. Traditional methods like linear regression and homogeneous transformation matrices (HTM) perform well in static compensation but struggle with time-varying and nonlinear coupling under dynamic conditions[343,344]. Recently, combinations of statistical analysis methods (such as PCA and independent component analysis, ICA) and digital twin technologies offer new approaches for multidimensional error data dimensionality reduction, decoupling, and online prediction.

PCA transforms original variables into orthogonal principal components, helping identify and prioritize major error sources; it is well-suited for decoupling multivariate data such as thermal drift and position errors[348]. By contrast, ICA decomposes multivariate signals into independent components, effectively identifying errors caused by factors such as thermal effects and vibration, and is especially suitable for dynamic and nonlinear error source analysis[349]. While PCA and ICA have significant advantages in error dimensionality reduction and decoupling, they must still be used in synergy with traditional physical modeling and digital twin systems to achieve a comprehensive breakdown and precise control of machine tool errors.

As manufacturing systems move deeply toward intelligence and digitalization, error decoupling technology is evolving from traditional "static constant-value compensation" to advanced "dynamic perception and adaptive control". Given the different physical mechanisms of various error sources, current research focuses on four key types: geometric error, thermal error, cutting force error, and dynamic error. Table 5 summarizes typical decoupling methods, key technical tools, and experimental validation means for these errors.

The research on geometric errors is shifting from static modeling to dynamic system modeling, and solutions for thermal errors are also upgrading from offline compensation to real-time temperature field prediction. At the same time, the research on cutting force errors is progressing from macro force monitoring to micro cutting state perception, while dynamic errors are evolving from single-frequency static correction to adaptive time-frequency domain control.

Table 5. Decoupling techniques and engineering verification paths for typical machine tool error types.

Error type	Core challenges	Decoupling methods	Core technology tools	Experimental verification methods	Engineering value	References
Geometric error	Multi-body coupling and nonlinear modeling	1. Multi-body system modeling (HTM) 2. Modal decoupling (ODS + MAC) 3. Sensitivity analysis (grey relational analysis)	Laser interferometer, DBB, NURBS surface fitting, finite element software (ANSYS/LMS)	Laser tracking for spatial geometric error measurement, static calibration, 3D trajectory testing with ball-bar	Improved positioning accuracy	[343,350,351]
Thermal error	Time-varying temperature field coupled with structural response	1. Error decomposition (spindle/rotary table/bed separation) 2. Optimization of temperature-sensitive points (grey relational analysis) 3. Nonlinear modeling (NLP)	Temperature arrays (PT100, thermocouples), infrared thermography, self-decoupling force/torque sensors, BP neural networks, SVR	Thermal cycling tests (40-day temperature tracking), temperature-deformation mapping validation, cutting heat drift compensation	Suppression of thermal deformation	[339,352,353]
Cutting force error	Multi-field coupling and state uncertainty	1. Force sensor fusion 2. Time-frequency domain analysis (wavelet transform) 3. Stiffness compensation optimization	Self-decoupling force/torque sensors, MEMS six-axis force sensors, cutting force prediction models (LSTM/Random forest), FEM simulation (ABAQUS)	Cutting tests with force-vibration coupling, tool tip monitoring, material removal rate vs. vibration threshold testing	Extended tool life	[354,355]
Dynamic error	High-frequency vibration and servo lag	1. Frequency domain decoupling (FFT + modal analysis) 2. Dynamic feedforward control (acceleration compensation) 3. Damping optimization (vibration damping devices)	Piezoelectric accelerometers, modal analysis instruments (LMS), adaptive PID controllers, DBB device	Modal impact testing, vibration spectrum analysis during machining, dynamic roundness tracking	Reduced surface roughness	[356,357]

Real-time decoupling of multi-source machining errors faces challenges due to their nonlinear, time-varying, and coupled properties, including high-dimensional error source interaction, dynamic working conditions, sensor synchronization issues, and limited computational resources[358]. For nonlinear coupled systems and time-varying conditions, researchers have proposed a data-based iterative dynamic decoupling control method specifically addressing coupling errors in precision MIMO motion systems[359]. Through the technical route of

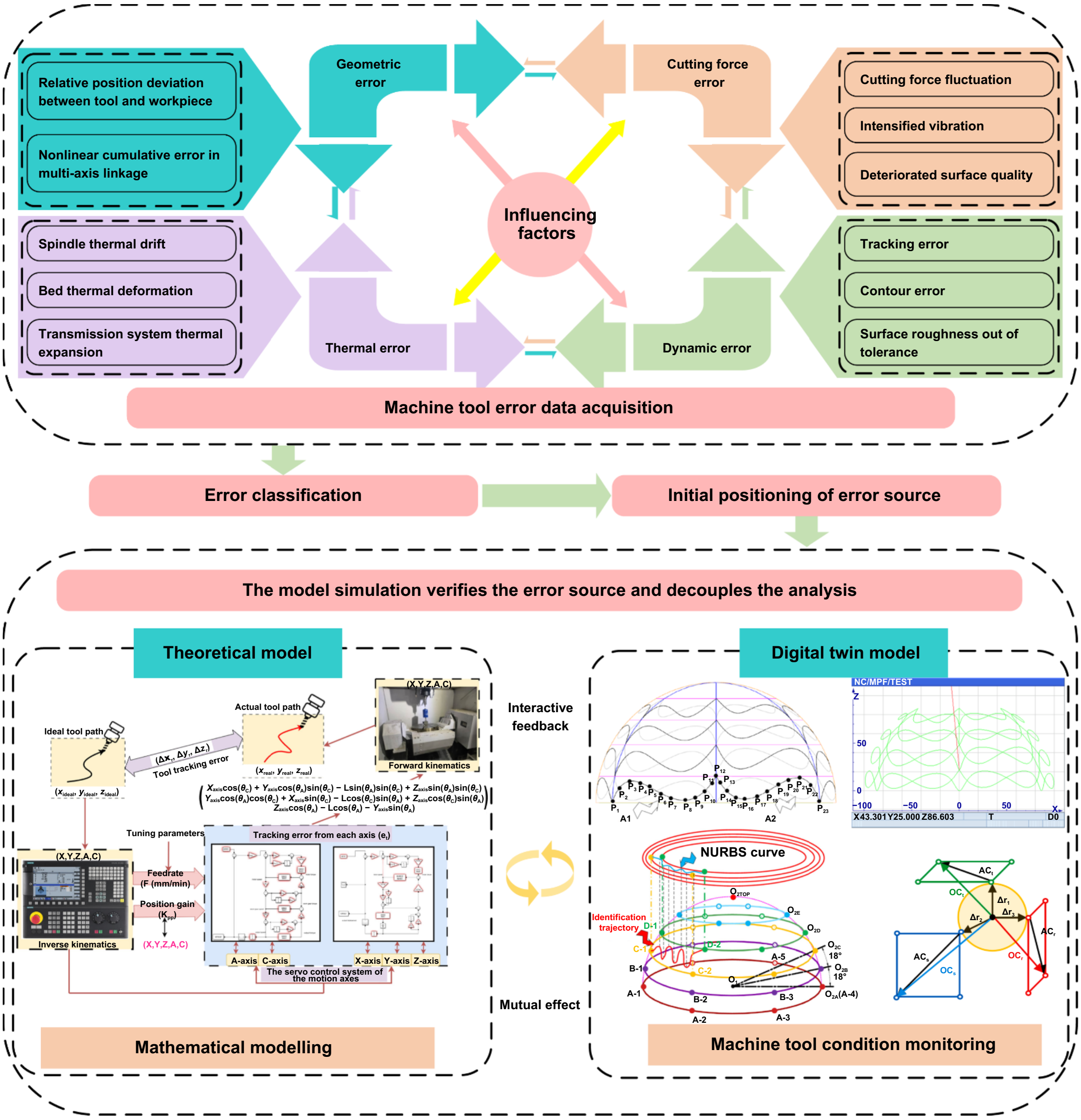

Figure 17. Machine tool error traceability and decoupling framework. Adapted from[351], with permission from Springer Nature. Adapted from[357], with permission from Springer Nature. Reprinted from[362], Copyright (2025), with permission from Elsevier.

"theoretical modeling—key variable selection—error component separation—engineering implementation", it effectively solves the problem of coupling identification between geometric error and thermal error in machining center linear axes[339]. Digital twin technology offers new approaches to these issues by supporting error evolution visualization, parameter self-update, and AI-enhanced decoupling through virtual replication and real-time synchronization, significantly enhancing error identification and control capabilities under complex conditions.

Therefore, researchers have proposed a digital twin–based multi-source error decoupling framework, integrating hybrid modeling, multi-source data perception, and multi-physics simulation, significantly improving system error traceability and control accuracy[360,361]. As shown in Figure 17, this framework comprises five main levels: (1) data acquisition layer, combining ODS modal decoupling, laser tracking, and multi-point sensors for high-quality dynamic error data collection[343,350,354]; (2) error classification layer, using PCA and wavelet transforms for signal dimensionality reduction and feature extraction to enhance error pattern separation accuracy[353,356]; (3) error-source localization layer, integrating finite-element thermal–structural coupled simulation and laser tracking techniques for quantitative identification and physical-position pinpointing of error sources[351,357]; (4) model decoupling layer, where multi-source sensor fusion is key for real-time error traceability and decoupling. By collecting complementary information across thermal, force, vibration, and other physical domains, and using hierarchical sequential memory and nonlinear regression modeling, a robust time-varying error model is constructed to enhance generalization ability and online modeling capability[339,352]; (5) real-time compensation layer, constructing a closed-loop control system via digital twin as the carrier, integrating feedforward correction and feedback compensation mechanisms to achieve dynamic suppression of system-level errors.

This framework provides theoretical support and an engineering paradigm for error traceability and adaptive control under complex working conditions, optimizing system dynamic response and steady-state accuracy, and laying the foundation for high-precision operation in intelligent manufacturing systems[363].

6. Error prediction

In recent years, data-driven prediction methods based on massive historical data and real-time sensor signals, as well as hybrid prediction strategies that integrate physical modeling with data analysis, have become key approaches to enhancing machine tool performance and enabling intelligent manufacturing. Error prediction focuses on forecasting future error trends based on current operating conditions, sensor feedback, and historical data. The goal is to enable feedforward intervention—actively controlling the process before errors affect critical dimensions—thereby improving the stability of the machining process.

6.1. Data-driven prediction method

The data-driven prediction method utilizes historical data and real-time signals to capture the complex variation patterns in machine tool operation, enabling efficient error prediction and real-time monitoring. Inspired by the interconnected structure of brain neurons, artificial neural networks (ANN) excel at describing nonlinear relationships between inputs and outputs, and are widely used for the prediction of thermal and geometric errors[364–366]. By training on historical processing data and sensor signals such as temperature and vibration, the ANN model can dynamically update its parameters, improving the prediction accuracy for various types of errors[367]. Support vector machines (SVM) construct optimal hyperplanes to effectively distinguish complex error patterns in high-dimensional spaces, making them particularly suitable for scenarios with complex data categories and mixed error patterns[368]. SVMs have significant advantages in capturing subtle changes in machine tool operation, enabling the detection of minute patterns that may precede specific errors.

By combining ANN, SVM, and IoT technology, the real-time performance and accuracy of error prediction can be further enhanced[369]. The IoT system collects real-time data such as temperature, vibration, and cutting force, and predicts and compensates for CNC machining trajectory errors through hybrid model processing. With an online learning mechanism, the prediction model iteratively updates as new data is input, gradually identifying more complex error patterns and continuously improving prediction accuracy[322]. This adaptive optimization not only effectively reduces error rates but also minimizes unplanned downtime through intelligent maintenance strategies, facilitating the development of manufacturing toward higher efficiency and intelligence[370].

In summary, data-driven methods such as ANN and SVM have made significant breakthroughs in the field of machine tool error prediction. These methods not only enable real-time monitoring and early warning of errors but also dynamically adapt to complex operating conditions, reducing production costs, enhancing system reliability and product quality, and driving the manufacturing industry toward greater intelligence and efficiency.

6.2. Physical-driven prediction method

The physics-driven prediction method provides a theoretical basis for error traceability and compensation by analyzing the response of machine tool structures under mechanical loads and thermal boundary conditions. Key technologies include finite element analysis (FEA), thermo-mechanical coupling simulation, and their integrated applications. These methods, combined with material properties and real-time monitoring, enable a comprehensive prediction process from design optimization to online control.

FEA discretizes complex structures into computational elements, simulating stress–deformation behavior under varied loads, constraints, and thermal effects. It precisely evaluates how component interactions affect overall accuracy and is well-suited for structural optimization and error prediction in high-precision machine tools[371]. Vibration instability (such as chatter) during machining is a key challenge in precision manufacturing. To predict chatter stability under various cutting conditions, researchers have developed numerous stability lobe diagrams (SLDs)[372]. For parallel or hybrid machine tools, consideration of passive node stiffness variation under load led to a structural dynamics model that reveals modal response behavior, achieving more accurate stability limit predictions for parallel tool heads in thin-wall machining[373]. Additionally, in milling thin-wall workpieces where material removal induces time-varying dynamics, a dual reduction strategy ("substructure partitioning + free interface" method) and boundary degree-of-freedom elimination overcame low computational efficiency and boundary-dependence issues of traditional methods[374].

Thermo-mechanical coupling analysis focuses on the impact of temperature-induced thermal expansion, contraction, and associated mechanical stresses on machine tool accuracy[375]. Through multi-physics field simulation, it can dynamically predict the evolution of thermal errors during the machining process[344]. Combining FEA with thermo-mechanical coupling analysis enables the construction of a multidimensional, comprehensive error prediction system[360,376,377]. The differences in thermal expansion coefficients, elastic moduli, and thermal conductivities of different materials directly affect the evolution of machine tool errors[378]. For example, cast iron, with its high thermal stability, is commonly used for manufacturing high-precision machine tool beds, while lightweight materials such as aluminum alloys or carbon fiber composites, although reducing weight, require additional temperature compensation due to their higher thermal expansion coefficients. Ceramic guides, known for their low friction and high thermal conductivity, are used to reduce thermal deformation[72]. Therefore, physics-based prediction methods must incorporate material characteristics and customize simulation parameters to improve the accuracy of error predictions.

By integrating IoT and intelligent sensing systems, real-time data such as temperature, vibration, displacement, and cutting force can be collected and fed back into the physical model, enabling dynamic prediction and error correction. Feedback loops based on machine learning techniques, such as random forests, further enhance the online correction of thermal errors and the optimization of machining parameters[233].

Physics-driven methods provide solid support for error traceability, design optimization, and real-time compensation by deeply revealing the interactions among mechanical stresses, thermal effects, and material properties. With the ongoing advancements in computational and sensing technologies, the integration of data-driven optimization will accelerate the development of high-precision, intelligent manufacturing systems.

6.3. Hybrid prediction method

The physics-driven approach reveals the interactions among thermal, force, and structural effects, offering good interpretability and extrapolation, but it requires significant computational resources and is hard to update online. Data-driven methods excel at capturing nonlinear features from sensor data, making them effective for anomaly detection in dynamic conditions, but they struggle to generalize to new conditions.

The hybrid framework combines both approaches, achieving high fidelity and adaptability for accurate prediction and real-time compensation of multi-source errors[61]. For example, in the context of thermal error modeling, researchers improved the performance of BP neural networks by adjusting the inertia weight coefficient in the particle swarm optimization algorithm and introducing an S-shaped function to enhance the backpropagation process. This significantly improved prediction accuracy, reaching 96.5%[367]. In addition, for fault diagnosis of spindle systems, physical models such as bearing wear and imbalance were embedded into a digital twin-based spindle simulation. The simulated data were used to train a GRU network, which was then fine-tuned with a small amount of experimental data, achieving a fault localization accuracy of 97.6%, significantly outperforming purely physical or purely data-driven models[379].

As shown in Figure 18, the hybrid prediction based on digital twins achieves deep integration of mechanism constraints and data-driven methods by constructing a bi-directional mapping between the physical and virtual systems. The process includes multi-physical data synchronization and filtering, feature extraction, joint modeling, and edge deployment to enable millisecond-level dynamic error correction and closed-loop control. This method has been widely applied in predicting geometric, thermal, and dynamic errors[364,380,381], and has shown excellent scalability in fields such as aerospace, high-speed cutting, and full-life-cycle health management. Current research is focused on time-series neural networks and multi-scale feature modeling to further enhance system response speed and prediction accuracy[322]. For example, in CNC milling processes, researchers combine power/cutting force predictions from physical models with residual signals from incremental data-driven models (such as decision trees or neural networks), and apply CUSUM (Cumulative Sum) testing for anomaly detection. Under a 1% noise background, this approach achieved a detection accuracy of 92%, effectively reducing false alarm rates[382]. In addition, to address sample scarcity and distribution mismatch in thermal error modeling, researchers used high-fidelity physical models to generate synthetic data. This was combined with a distance-guided domain adversarial network and maximum mean discrepancy (MMD) metrics to enable cross-domain transfer learning of thermal error data, achieving an 11.73% improvement in goodness of fit. This method proves suitable for high-accuracy prediction under small-sample conditions[40].

Despite significant progress in hybrid prediction methods, several challenges persist in actual dynamic machining environments. Firstly, rapidly changing machining parameters and highly coupled error sources increase the complexity of modeling. Secondly, multi-source sensor data may suffer from temporal asynchrony and unobservable latent variables, which raise model uncertainty. Furthermore, adaptive update mechanisms can be disturbed under abrupt operating conditions, and high computational loads limit system response speed. Therefore, achieving stable and efficient real-time prediction still requires continuous breakthroughs in modeling accuracy, data synchronization, and computational efficiency.

Future work should develop an integrated architecture combining mechanistic modeling, data-driven approaches, and intelligent decision-making; leverage nano-scale monitoring, distributed digital twins, and explainable AI; improve model transparency and adaptivity; overcome bottlenecks in real-time prediction and intelligent control; and accelerate practical deployment on the factory floor[385,386].

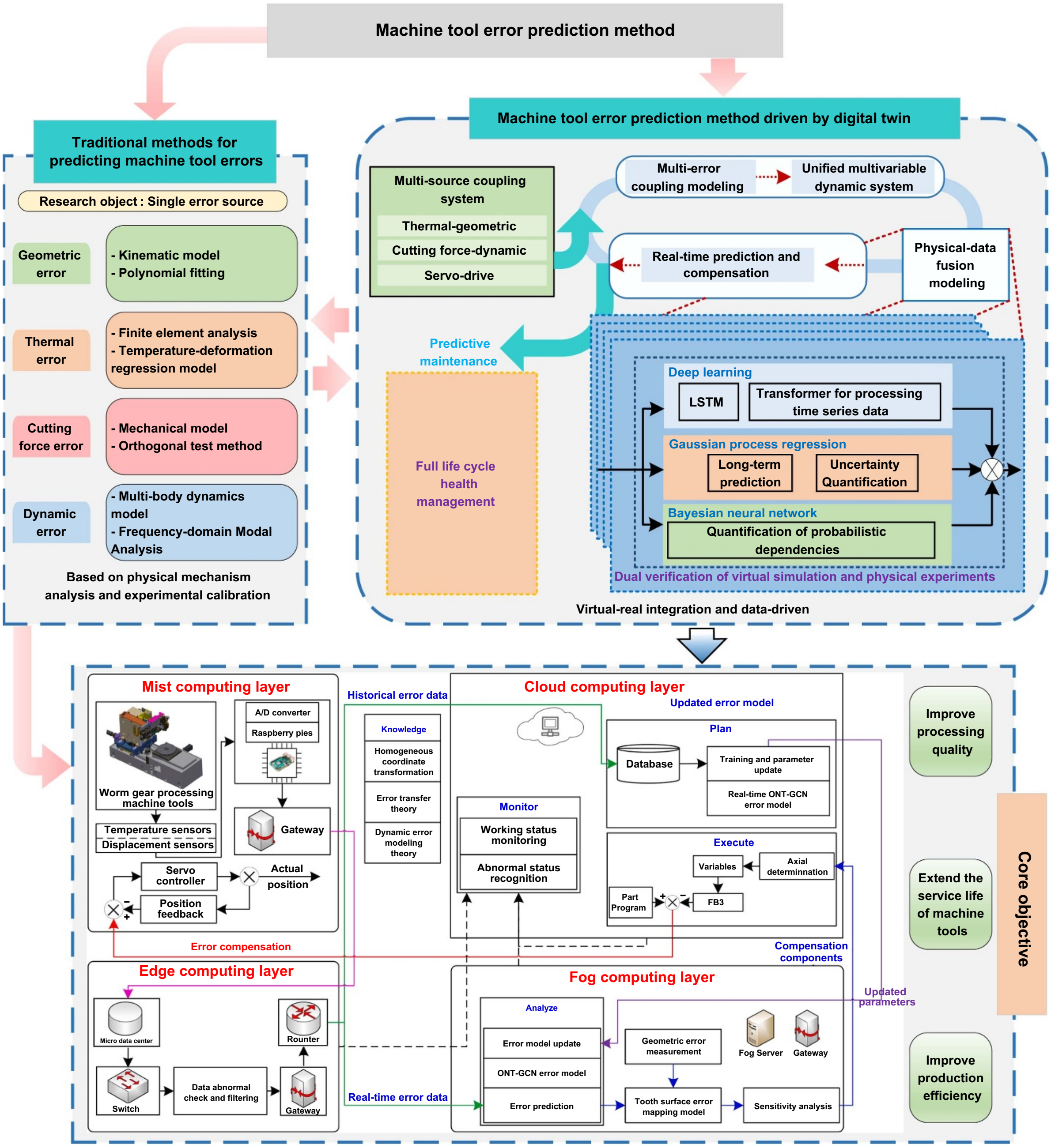

Figure 18. Research progress of error prediction[383,384]. Reprinted from[384], Copyright (2023), with permission from Elsevier.

7. Compensation strategy

Error compensation employs adaptive feedback control mechanisms to make real-time corrections by dynamically adjusting servo parameters, thermal control systems, or tool paths. This module directly determines the final machining accuracy, aiming to minimize dimensional errors, surface defects, and stability fluctuations.

7.1. Geometric error compensation

Machine tool geometric errors are a key factor affecting machining accuracy. To ensure product quality, efficient compensation strategies must be adopted. Currently, error mapping and real-time compensation technologies are widely used and continuously evolving, forming the core of geometric error control.

Error mapping techniques systematically measure and quantify multi-dimensional geometric errors of machine tools, such as linear positioning error, angular error, straightness error, and perpendicularity error[387]. Common measurement devices include laser interferometers, ball bar instruments, and coordinate measuring machines, which generate error distribution maps that support error modeling and compensation. These maps not only reveal the spatial distribution of errors but also provide a basis for targeted compensation strategies[388].

Given that errors evolve over time due to mechanical wear and temperature changes, it is necessary to periodically update these maps to ensure the effectiveness of compensation.

On the basis of mapping, real-time compensation techniques enable dynamic error control. By deeply integrating error models with CNC systems, these technologies can acquire machine status in real time and perform error prediction, then dynamically adjust tool paths or process parameters. For example, an online surface metrology system based on an optical slope sensor can provide error feedback and synchronize parameter adjustment during precision grinding, effectively reducing form errors[389]. Achieving efficient real-time compensation requires high-resolution sensors, high-performance CNC systems, and integrated predictive models and AI algorithms to enhance error prediction and feedforward control capabilities.

Spatial error compensation algorithms within CNC systems, such as pitch error compensation and sag compensation, have been widely applied in industry. Pitch compensation is primarily used to correct axial positioning errors, while sag compensation addresses non-axial deformations of the machine structure caused by gravity. The synergistic effect of these two methods effectively enhances the spatial motion accuracy of machine tools. By integrating error mapping with real-time compensation techniques, not only can machining accuracy and repeatability be significantly improved, but the application scope can also be extended—from three-axis machines to five-axis systems[234].

With the advancement of vision measurement and intelligent calibration technologies, geometric error compensation is becoming more intelligent. Combining high-resolution CCD cameras with sub-pixel image processing makes error quantification more intuitive. The error calibration system based on CCD visual measurement and global image registration can precisely calibrate the geometric errors of the two-dimensional precision platform and achieve nanometer-level positioning compensation[390]. Additionally, self-calibration systems based on laser galvanometer scanning, combined with Kalman filtering and machine vision, have successfully reduced positioning error to ± 1 μm, verifying the potential of intelligent algorithms in high-precision error control[391].

In summary, error mapping and real-time compensation represent a deep integration of traditional precision measurement and modern intelligent control and remain the core direction of geometric error control. However, achieving efficient real-time compensation in high-speed, multi-axis systems still faces technical challenges: adaptive compensation modules must handle continuous changes in axial speed and acceleration under complex trajectories, ensuring dynamic matching of correction amounts and continuous motion without exciting structural resonances; current industrial controllers also lack sufficient parallel computation power to support six-degree-of-freedom error compensation, servo control, thermal drift correction, and dynamic error suppression simultaneously. With the ongoing development of high-performance sensors, edge computing platforms, and AI, geometric error compensation is expected to advance toward higher intelligence, integration, and precision—promoting sustained precision retention in high-end manufacturing.

7.2. Thermal error compensation

Thermal errors in machine tools mainly arise from structural thermal expansion and uneven temperature gradients during operation, directly causing relative pose shifts between the tool and workpiece. While thermal symmetry design, insulating materials, and thermal equilibrium layouts can partially mitigate thermal effects, the physical nature of heat conduction and material expansion makes complete elimination impossible. Therefore, thermal error compensation is key to maintaining machining accuracy.

Early thermal error compensation relied on mechanical rigidity correction—such as the pitch correction screw bar used in the 1950s—that addressed a single error source through static correction and lacked adaptability[392]. With CNC development, coordinate-offset-based thermal error compensation became mainstream, introducing corrective displacements in CNC commands to economically and effectively negate thermal errors online[393]. Current compensation schemes typically follow three core steps: modeling/prediction, measurement-point optimization, and real-time compensation, emphasizing the synergistic optimization of modeling accuracy and online responsiveness.

In modeling and prediction, existing methods fall into two categories: empirical statistical models and physics-based models. Empirical models using least-squares fitting and modular compensation systems can achieve high predictive accuracy and good system compatibility[394]. Self-organizing feature maps and improved particle-swarm-optimized neural networks can enhance nonlinear fitting and generalization performance in thermal error modeling[367]. In contrast, physics-based models grounded in heat conduction principles and material thermodynamics more authentically capture thermal field distribution and induced deformation effects. For example, dynamic modeling and compensation methods from a system-identification perspective can eliminate pseudo-hysteresis influences in traditional models and have been validated by spindle testing[395]. A piezo-driven real-time thermal error compensation system for spindle transient thermal elongation enables closed-loop control under high-speed rotation[396].

Measurement-point optimization is crucial for improving model efficiency and prediction stability. Correlation analysis identifies key measurement points strongly related to thermal errors, reducing data redundancy. Techniques such as direct criterion and indirect grouping can compress input variable counts, reduce mutual coupling, and shorten modeling time[397]. Analyzing sensitivity variation of temperature points with operating conditions and applying principal component regression can mitigate multicollinearity and further improve prediction accuracy and compensation performance[398]. Greyscale relevance models optimizing measurement layouts can reduce temperature inputs from 16 to 4 points, ensuring modeling accuracy and enhancing computational efficiency[399].

Real-time compensation requires high-speed data acquisition and dynamic control. Integrating temperature and vibration sensors into self-optimizing machining systems allows dynamic adjustment of feed rate and cutting parameters based on real-time conditions, effectively reducing coupling between thermal errors and mechanical wear[68]. Applying machine learning for online thermal error prediction on five-axis machine tools and combining it with error mapping to correct tool paths in real time can reduce machining errors by 85%[233]. Active thermal control plates combined with model-driven power-matching strategies also provide effective means for managing thermal distribution and enhancing thermal stability.

Despite significant progress in current research, thermal error compensation still faces several challenges: first, the rapid identification of thermally sensitive measurement points in complex structures and poor dynamic adaptability; second, the need for further research to develop predictive models with strong robustness and generalization capabilities; third, the integration mechanisms of multi-physics field simulations and models are not yet mature; fourth, balancing computational complexity and model accuracy while ensuring real-time performance; and fifth, the trade-off between compensation system implementation cost and industrial adaptability needs to be addressed. Systematic research on these issues will provide key technical support for achieving high-precision and high-efficiency machine tool thermal error compensation and thermal management.

7.3. Dynamic error compensation

To achieve high machining accuracy and stability, dynamic error compensation technologies increasingly focus on vibration suppression, adaptive control, and model-based closed-loop correction, emphasizing real-time error perception and rapid response capability.

Vibration suppression methods include passive and active strategies. Passive methods use dampers and isolating structures to reduce vibration transmission without complex control systems. Examples include integrating hydraulic dampers to significantly suppress spindle–workpiece resonance and improve stability in titanium alloy machining[400]; using magnetorheological dampers to control chatter in flexible workpieces with good adaptability for small-batch, multi-variety production[401]; and optimizing viscoelastic damper parameters based on fractional-order derivative models to suppress low-frequency residual vibration, reducing machining error by 60%[402].

Active vibration suppression builds a closed-loop feedback system with sensors and actuators to monitor vibrations in real time and apply counterforces, offering higher response speed and adaptability. For instance, combining accelerometers with servo drives and using feedforward control to dynamically adjust feed rate can suppress cutting force fluctuations by 50% and significantly extend tool life[403]. While active systems are more costly and complex, their advantages make them indispensable for improving machining precision and stability in high-end manufacturing.

Adaptive control technology dynamically adjusts machining strategies by online sensing of working condition parameters such as thermal deformation, load variations, and vibrations, thereby enhancing machining consistency and robustness[404]. In multi-axis simultaneous machining, to address contour errors caused by servo lag, inter-axis synchronization errors, and dynamic structural deformation, researchers have proposed various real-time compensation schemes. For example, a control method based on trajectory geometry analysis and axis decoupling design avoids the complexity of traditional Jacobian inversion operations and effectively suppresses control chatter[405].

In recent years, model-driven control methods have gradually become dominant in dynamic error compensation. By constructing machine tool dynamic models and error prediction algorithms, real-time correction of machining paths can be achieved. For example, using model error compensation combined with an Efficient Contour Estimation Algorithm (ESA) can reduce the maximum contour error of five-axis machine tools by up to 66.8%[406]. For thin-walled part machining, a multi-channel predictive compliance model incorporating residual stress monitoring enables the coordinated optimization of tool paths and cutting parameters, significantly suppressing deformation[407].

AI and ML integration bring new breakthroughs to dynamic error compensation. Deep learning models can mine error evolution patterns from historical data and enable early warning and adaptive correction. For example, combining LSTM networks with system identification to build iterative error prediction and compensation mechanisms improved model accuracy and computation efficiency in five-axis machining[408]. Digital twin-based compensation systems that fuse cutting force, temperature, and multi-source data enable dynamic parameter optimization and enhance deformation control for complex structural parts[326].

In summary, dynamic error compensation is evolving from traditional static control to multi-level closed-loop systems that integrate model-driven, data-driven, and intelligent algorithmic methods. Going forward, leveraging advanced sensors, intelligent control algorithms, and edge computing platforms, this technology will play a central role in ensuring machining accuracy and system stability, providing a solid foundation for intelligent manufacturing.

7.4. Integrated error compensation system

With the ongoing rise in high-precision manufacturing requirements, traditional error compensation methods that rely on single sensors or empirical models can no longer cope with the challenges posed by multi-source error coupling, nonlinearity, and time-varying behavior. As a result, developing integrated error compensation systems with real-time sensing, dynamic modeling, and closed-loop control capabilities has become a key trend in precision manufacturing.

Modern integrated systems combine high-precision sensors, edge computing platforms, and multi-source modeling methods to enable full-lifecycle error management and dynamic optimization. The integration of measurement and compensation supports dynamic calibration techniques, providing robust support for long-term high-precision operation[409].

As shown in Figure 19, the integrated error compensation architecture summarized in this paper includes core modules such as multi-source error sensing and real-time data fusion, multi-source error modeling and prediction, dynamic feedback and closed-loop control, as well as real-time compensation and visualization. This architecture relies on a digital twin platform to achieve dynamic synchronization between the physical entity and its virtual model, thereby enabling predictive and feedforward error compensation mechanisms.

Compared with traditional systems, this architecture places greater emphasis on coordinated regulation among error sources and real-time interaction among system modules. For example, by jointly modeling differential models and thermal imaging, and applying a weighted least squares fusion strategy, residual errors were reduced to 0.3 μm—significantly outperforming the results of single-model approaches[414]. Under complex working conditions, a physical-statistical hybrid model has been used to realize stepwise thermal error compensation[415]. Furthermore, by integrating tool rotation error, geometric error, and cutting-force-induced elastic deformation, surface roughness prediction error can be effectively controlled through point cloud analysis and closed-loop sensor feedback[416].

In addition, for thermal error modeling under multiple heat sources, researchers have adopted a combination of improved Dempster–Shafer (D-S) evidence theory and radial basis function (RBF) neural networks, achieving a prediction accuracy as high as 98.8%[417]. For comprehensive compensation of dynamic and static errors, related studies have shown that static errors can be transmitted through differential motion matrices, while dynamic errors are corrected online using machine learning models, thereby constructing a high-fidelity digital twin compensation core[49].

The system integrates multi-source signals such as spindle temperature, servo current, cutting force, and structural vibration, enabling real-time adjustment of key parameters (e.g., thermal expansion coefficient, stiffness matrix), and implements fast response and model optimization through a hierarchical feedback mechanism: the fast layer achieves sub-millisecond response, while the slow layer performs strategy correction and adaptive updates. In the face of abnormal conditions such as tool breakage or sensor drift, the system can automatically trigger self-calibration procedures to maintain robustness and stability. For example, servo lag and contour error compensation algorithms can adaptively adjust to balance tracking accuracy and vibration suppression performance.

In recent years, hybrid modeling and ensemble learning have been widely applied to contour error prediction and compensation, effectively addressing precision problems caused by data sparsity and insufficient real-time performance in commercial CNC systems[34]. Meanwhile, the composite method combining offline pre-compensation and online feedback control has effectively overcome the bottlenecks of real-time and accuracy in five-axis machining contour error control[418].

With the development of the IoT and edge computing, integrated compensation systems are rapidly evolving toward interdisciplinary integration and intelligence. These systems not only provide continuous state monitoring and adaptive maintenance capabilities, but also enable dynamic adjustment of machining parameters and self-optimization control, thereby supporting high-precision operation of machine tools throughout their entire life cycle[229,419].

At present, integrated compensation strategies have been widely applied across multiple dimensions, including geometric error, thermal error, cutting-force error, contour error, and time-varying error[41,322,326]. For instance, by establishing dynamic feature models of thin-walled parts, researchers obtained real-time correlations among workpiece stiffness, geometric state, and milling force. Based on this, a real-time compensation method for deformation error was proposed, which controlled workpiece thickness error within 10%[420]. Meanwhile, the GTF method that combines grey relational analysis, thermal sensitivity analysis, and fuzzy C-means clustering significantly reduced collinearity and information loss risks in thermal error modeling of multi-heat-source machine tools. Experiments showed that the average reductions in root mean square error were 28.0% and 25.8%, respectively, demonstrating strong generality and compensation performance[421].

Driven by AI systems, sensing and modeling technologies for real-time multi-source error compensation have seen continuous breakthroughs. A novel thermal error sensing system, which combines non-contact temperature measurement and principal component analysis, has achieved submicron control accuracy for the Z-axis thermal error of the spindle, providing a theoretical foundation and technical support for self-sensing and self-compensation in intelligent machine tools[417]. In addition, researchers have constructed a hybrid-model-based CNC digital twin framework, using AI algorithms to achieve trajectory error prediction and adaptive compensation, validating the feasibility and effectiveness of this method in terms of prediction accuracy and control performance[322].

The integrated compensation system demonstrates significant advantages in multidimensional error management through dynamic prediction and real-time control. Based on an interactive spatio-temporal graph convolutional network (ST-GCN) and a cloud-edge collaborative digital twin architecture, the system efficiently fuses temporal and spatial error information, achieving an optimal balance between micron-level compensation accuracy and millisecond-level response speed. Compared to traditional LSTM and Transformer models, this system improves prediction accuracy by 76%–88%, reaches a model fitting degree (R^2) of 98.7%, and reduces machining errors by an average of about 90%[422]. In hybrid models, physical models are used for data preprocessing or setting constraints[423], while machine learning, deep learning, and transfer learning are used for real-time parameter adjustment[424,425]. In industrial applications, hybrid models

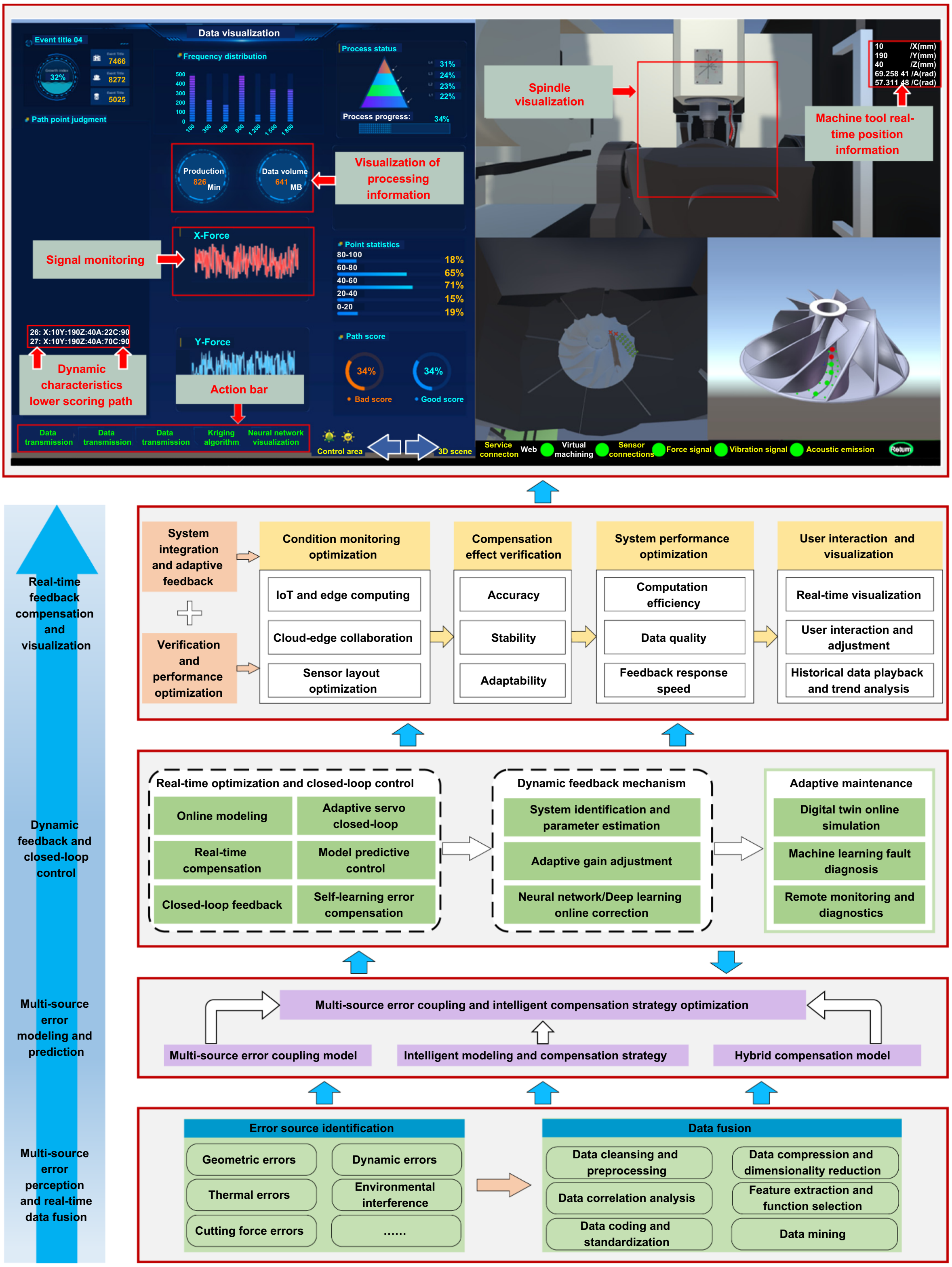

Figure 19. Framework of integrated error compensation system[282,326,410–413]. Reproduced from[326].

are used to adjust machine tool operations based on sensor data for real-time compensation, thus improving machining accuracy[426,427].

Another study combined a TCN-BiLSTM model with a G-code interpreter to build a customized compensation system capable of predicting X- and Y-axis tracking errors, achieving mean absolute errors as low as 0.000 009 mm and 0.000 023 mm, respectively. In complex trajectory machining (e.g., circular and heart-shaped paths), the system reduced axial errors by 45%–75% and 40%–70%, respectively, significantly enhancing dynamic control performance in ultra-precision machining. A cloud-training–edge-inference structure enabled millisecond-level response, and the system demonstrated robustness and industrial scalability across 12 transfer tests[409].

AI algorithms can identify residual errors and dynamically adjust compensation parameters to cope with tool wear and thermal load changes. Big data technologies mine historical machining information to reveal error evolution patterns. Digital twins integrate virtual simulation with real-time feedback to achieve error prediction and pre-compensation, thereby enhancing overall system performance.

However, intelligent compensation still faces challenges such as managing heterogeneous high-frequency data, limited model generalization capabilities, high complexity in cloud-edge coordination, poor model interpretability, and a lack of standardized interfaces. Future research should focus on optimizing algorithms and system architecture, promoting standardization, and accelerating the engineering application of intelligent compensation in precision manufacturing.

In summary, the development of integrated error compensation systems needs to advance in the following directions: construct open and high-performance CNC platforms with standardized interfaces to enhance system compatibility and scalability; integrate data from multiple axes, tools, and materials to establish a multi-source modeling framework with generalization capability; optimize modeling and control algorithms to strengthen real-time performance and robustness; deepen the integration of software and hardware, promote industrial demonstration applications, and accelerate the real-world deployment of intelligent compensation systems.

8. Conclusions

Current research on machine tool error control mostly focuses on modeling and compensating for single error sources, lacking systematic integration and engineering solutions that address nonlinear coupling, dynamic evolution, and closed-loop regulation of multi-source errors. To fill this gap, this paper constructs a systematic multi-source error evaluation framework for machine tool accuracy retention, covering the entire process of error identification, modeling, traceability and decoupling, prediction, and compensation. An integrated control architecture centered on digital twins and AI is developed, featuring online sensing, integrated modeling, adaptive prediction, and closed-loop compensation capabilities, achieving full-process optimization from error perception to compensation execution. The study shows that integrated strategies significantly improve the accuracy and response speed of error management, providing critical support for design optimization, accuracy assurance, and intelligent operation and maintenance of high-end manufacturing equipment.

Firstly, this work systematically classifies and quantitatively evaluates various errors based on their formation mechanisms and key influencing factors, clarifying the dominant error sources. By combining measurement methods such as laser interferometers, ball bars, and vision systems, the performance differences between online and offline measurement technologies regarding real-time capability and environmental adaptability are compared, emphasizing the practical value of intelligent systems in improving evaluation effectiveness in industrial scenarios.

In error modeling, a systematic review is given of geometric errors based on homogeneous transformation and screw theory models, thermal errors using empirical and physical modeling methods, and dynamic errors through modal analysis and transfer function approaches. It is pointed out that traditional methods have limitations in real-time performance and generalization under high-speed and heavy-load conditions. Therefore, this paper focuses on integrated modeling methods that fuse physical modeling with data-driven approaches based on intelligent systems. This method achieves dynamic updating and high adaptability of error models through multi-source data fusion and virtual-physical synchronization mechanisms, improving modeling accuracy and robustness under complex operating conditions.

Regarding traceability and decoupling of multi-source coupled errors, this paper reviews multivariate statistical analysis methods such as PCA and ICA, as well as sensitivity analysis and error budgeting strategies, enabling effective identification of key error sources and separation of their action paths, thus providing theoretical support for key factor extraction and optimization.

In error prediction, artificial neural networks, support vector machines, finite element simulation, and thermo-mechanical coupled modeling strategies are compared. It is noted that pure physical models are limited by modeling complexity and real-time response, while data-driven methods may face issues of poor generalization. The paper summarizes hybrid prediction methods that integrate prior physical models with data-driven optimization mechanisms, which not only enhance the dynamic simulation ability of error evolution under different working conditions but also improve model adaptability and prediction accuracy, providing decision support for proactive maintenance and process adjustment.

In error compensation and control, typical methods such as pitch error compensation, real-time coordinate correction, active temperature control, vibration suppression, and adaptive control are summarized. The paper highlights intelligent compensation systems that integrate AI and digital twins. These systems enable closed-loop detection and dynamic control of multi-source errors, significantly improving machining accuracy and dynamic stability of machine tools.

Particularly, this paper emphasizes the key role of digital twins in the entire error control process. Through virtual-physical synchronization, multi-source sensing, and AI adaptive mechanisms, an intelligent control framework supporting full lifecycle management is constructed, promoting the shift of error control from "passive compensation" to "active optimization".

In summary, the multi-source error evaluation and control system constructed in this paper possesses real-time capability, adaptability, and scalability, providing technical support for accuracy assurance, stable operation, and intelligent maintenance of high-end manufacturing equipment. Looking forward, with the continuous advancement of intelligent manufacturing technologies, this integrated strategy will play an increasingly critical role in achieving ultra-precision control, extending equipment lifespan, and improving production efficiency, thereby facilitating the autonomous controllability and sustainable development of high-end manufacturing equipment.

9. Challenges and prospects

Although rapid evaluation and compensation technologies for machine tool accuracy retention have made continuous progress in recent years, due to the system's high coupling and complexity, multi-source errors generally exhibit strong nonlinearity, dynamic variation, and multi-scale coupling characteristics. Existing methods still face the following key challenges in engineering practice:

First, the dynamic adaptability of high-fidelity multi-physics modeling is insufficient. Current error modeling often relies on simplified assumptions, making it difficult to accurately characterize nonlinear coupled behaviors such as thermal-mechanical-dynamic interactions under real working conditions, especially in complex tool-workpiece-fixture systems. Meanwhile, models have poor adaptability to new structures and control systems and are prone to failure with changing conditions. Although hybrid modeling integrates the advantages of physical and data-driven approaches, it is still limited by high computational costs and insufficient model generalization capability, making continuous online learning and adaptive updating difficult to achieve.

Second, high-frequency sensing and real-time processing capabilities remain inadequate. Although high-precision sensors can acquire rich state information, high-frequency and multidimensional data pose stringent requirements on edge-cloud collaborative architectures, network bandwidth, and computing resources. Achieving millisecond-level preprocessing, multi-source fusion, and error identification in resource-constrained environments remains a technical bottleneck for improving online compensation accuracy and stability.

Third, there is a lack of unified standards for error evaluation and compensation. Currently, no standardized system exists for measurement references, evaluation metrics, and implementation procedures, resulting in poor comparability among different methods and limited cross-platform migration and engineering applications. It is urgent to establish unified and generalizable error evaluation and control specifications.

Facing the future development of intelligent manufacturing, machine tool error management is rapidly advancing toward a new stage of "multi-technology integration—virtual-physical collaboration—closed-loop optimization". As shown in Figure 20, to overcome the above bottlenecks, future research should focus on the following eight directions:

(1) AI-driven intelligent modeling and adaptive compensation: build predictive models with generalization capability and physical interpretability based on deep learning, transfer learning, and reinforcement learning, mining multi-source error information; realize real-time control and accuracy maintenance under complex working conditions through intelligent strategies.
(2) Deep integration of sensors and the IoT: deploy highly responsive perception modules at the machine tool edge to achieve sub-millisecond error detection; combine cloud-based model training and strategy updates to construct an integrated perception-decision-execution intelligent closed-loop system.
(3) Cross-disciplinary integration promoting innovation in error control models: integrate multi-physics modeling and experimental validation of geometric, thermal, force, and dynamic fields, combined with data-driven methods, to systematically reveal error coupling and propagation mechanisms, constructing high-fidelity model systems adaptable to the full lifecycle and capable of online updating.
(4) Application of new materials and advanced manufacturing processes: utilize low thermal expansion, high wear-resistant materials, and additive manufacturing processes to reduce structural deformation and wear errors at the source, enhancing machine tool stability and longevity.
(5) Standardized error evaluation and compensation protocols: establish unified standards for error identification, evaluation, and compensation to promote cross-platform and cross-machine general implementation specifications, improving system compatibility and technology transferability.
(6) Development of highly integrated and intelligent systems: combine multi-physics simulation, hybrid modeling, and intelligent optimization technologies to build closed-loop accuracy assurance systems with self-perception, self-learning, and self-adjustment capabilities, achieving goal-oriented active regulation.
(7) Multi-source data fusion and edge-cloud collaborative control: introduce advanced algorithms for deep fusion of heterogeneous data such as temperature, displacement, and vibration; enable rapid response at the edge and optimized decision-making in the cloud to enhance system robustness and generalization.
(8) Cross-platform collaboration and human-machine interaction: promote system deployment in small and medium-sized manufacturing enterprises based on standardized interfaces and low-cost sensors; build efficient and user-friendly human-machine collaboration modes through visualization interfaces and intelligent interaction technologies.

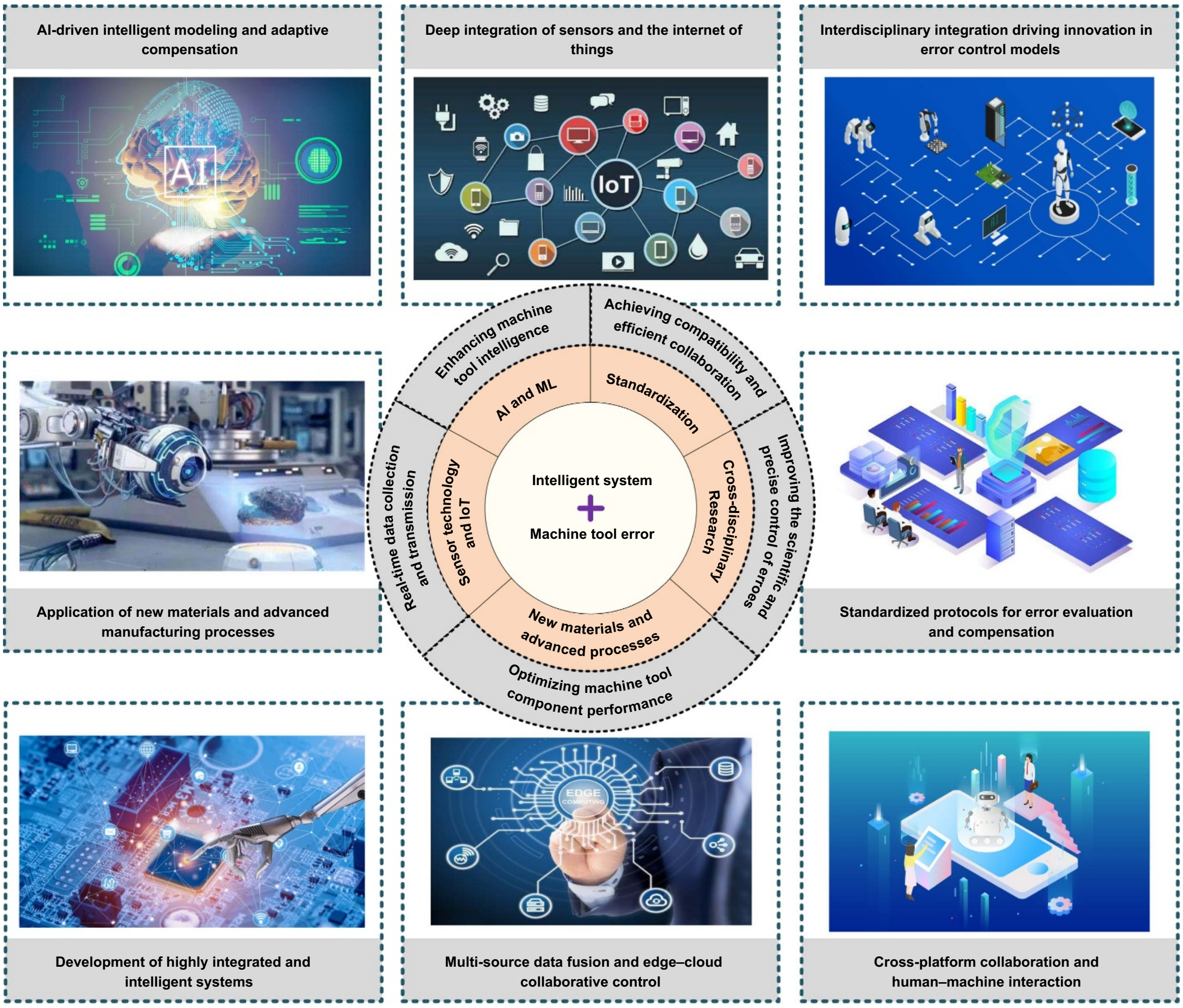

Figure 20. Prospects for research on multi-source errors of machine tools.

In summary, machine tool accuracy retention technology is progressing toward “multi-source fusion, virtual-physical collaboration, real-time response, and autonomous optimization”. Relying on key technologies such as digital twins, AI, and edge-cloud computing, it is expected to build an intelligent error management system oriented to the full lifecycle, empowering high-quality and intelligent manufacturing.

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References

[1] Liu C H, Jywe W Y, Jeng Y R, Hsu T H and Li Y T. 2010. Design and control of a long-traveling nano-positioning stage. *Precis. Eng.* **34**, 497–506.
[2] Lee K I and Yang S H. 2013. Accuracy evaluation of machine tools by modeling spherical deviation based on double ball-bar measurements. *Int. J. Mach. Tools Manuf.* **75**, 46–54.
[3] Wan M, Zhang W H, Qin G H and Wang Z P. 2008. Strategies for error prediction and error control in peripheral milling of thin-walled workpiece. *Int. J. Mach. Tools Manuf.* **48**, 1366–1374.
[4] Duan Z J, Wang S S, Wang Z H, Li C H, Li Y H, Song J L, Liu J Y and Liu X. 2024. Tool wear mechanisms in cold plasma and nano-lubricant multi-energy field coupled micro-milling of Al-Li alloy. *Tribol. Int.* **192**, 109337.

[5] Duan Z J et al. 2021. Milling Force model for aviation aluminum alloy: academic insight and perspective analysis. *Chin. J. Mech. Eng.* **34**, 18.
[6] Hu F, Zou X M, Hao H M, Hou P and Huang Y. 2024. Research and application of simulation and optimization for CNC machine tool machining process under data semantic model reconstruction. *Int. J. Adv. Manuf. Technol.* **132**, 801–819.
[7] Lee C Y, Hwang S H and Min B K. 2022. Simulation-based optimization of CNC interpolator parameters for machining accuracy improvement. *Int. J. Adv. Manuf. Technol.* **119**, 3757–3770.
[8] Yang J X and Ding H. 2016. A new position independent geometric errors identification model of five-axis serial machine tools based on differential motion matrices. *Int. J. Mach. Tools Manuf.* **104**, 68–77.
[9] Zhong G Y, Wang C Q, Yang S F, Zheng E L and Ge Y Y. 2015. Position geometric error modeling, identification and compensation for large 5-axis machining center prototype. *Int. J. Mach. Tools Manuf.* **89**, 142–150.
[10] Huang Z Q, Shen Y, Li J Y, Fey M and Brecher C. 2021. A survey on AI-driven digital twins in Industry 4.0: smart manufacturing and advanced robotics. *Sensors* **21**, 6340.
[11] Chen Y P, Gao J, Deng H X, Zheng D T, Chen X and Kelly R. 2013. Spatial statistical analysis and compensation of machining errors for complex surfaces. *Precis. Eng.* **37**, 203–212.
[12] Mo F, Rehman H U, Monetti F M, Chaplin J C, Sanderson D, Popov A, Maffei A and Ratchev S. 2023. A framework for manufacturing system reconfiguration and optimisation utilising digital twins and modular artificial intelligence. *Robot. Comput.-Integr. Manuf.* **82**, 102524.
[13] Friederich J, Francis D P, Lazarova-Molnar S and Mohamed N. 2022. A framework for data-driven digitial twins of smart manufacturing systems. *Comput. Ind.* **136**, 103586.
[14] Wang X Z, Wang Y C, Tao F and Liu A. 2021. New paradigm of data-driven smart customisation through digital twin. *J. Manuf. Syst.* **58**, 270–280.
[15] Wu Q C, Mao Y S, Chen J X and Wang C. 2021. Application research of digital twin-driven ship intelligent manufacturing system: pipe machining production line. *J. Mar. Sci. Eng.* **9**, 338.
[16] Fan Y P, Yang J Z, Chen J H, Hu P C, Wang X Y, Xu J C and Zhou B. 2021. A digital-twin visualized architecture for flexible manufacturing system. *J. Manuf. Syst.* **60**, 176–201.
[17] Luo W C, Hu T L, Ye Y X, Zhang C R and Wei Y L. 2020. A hybrid predictive maintenance approach for CNC machine tool driven by Digital Twin. *Robot. Comput.-Integr. Manuf.* **65**, 101974.
[18] Yang X, Ran Y, Zhang G B, Wang H W, Mu Z Y and Zhi S G. 2022. A digital twin-driven hybrid approach for the prediction of performance degradation in transmission unit of CNC machine tool. *Robot. Comput.-Integr. Manuf.* **73**, 102230.
[19] Yu H Y, Yu D, Wang C T, Hu Y and Li Y. 2023. Edge intelligence-driven digital twin of CNC system: architecture and deployment. *Robot. Comput.-Integr. Manuf.* **79**, 102418.
[20] Huang Y, Lu B H and Zhao W H. 2018. Application of cloud computing in intelligent machine tool control system. *J. Mech. Eng.* **54**, 210–216.
[21] Mihai S et al. 2022. Digital twins: a survey on enabling technologies, challenges, trends and future prospects. *IEEE Commun. Surv. Tutor.* **24**, 2255–2291.
[22] Erkoyuncu J A, Del Amo I F, Ariansyah D, Bulka D, Vrabic R and Roy R. 2020. A design framework for adaptive digital twins. *CIRP Ann.* **69**, 145–148.
[23] Glatt M, Sinnwell C, Yi L, Donohoe S, Ravani B and Aurich J C. 2021. Modeling and implementation of a digital twin of material flows based on physics simulation. *J. Manuf. Syst.* **58**, 231–245.
[24] Zheng P and Sivabalan A S. 2020. A generic tri-model-based approach for product-level digital twin development in a smart manufacturing environment. *Robot. Comput.-Integr. Manuf.* **64**, 101958.
[25] Wang S, Lai X N, He X W, Qiu Y M and Song X G. 2022. Building a trustworthy product-level shape-performance integrated digital twin with multifidelity surrogate model. *J. Mech. Des.* **144**, 031703.
[26] Lee J, Lapira E, Bagheri B and Kao H A. 2013. Recent advances and trends in predictive manufacturing systems in big data environment. *Manuf. Lett.* **1**, 38–41.
[27] Cerrone A, Hochhalter J, Heber G and Ingraffea A. 2014. On the effects of modeling as-manufactured geometry: toward digital twin. *Int. J. Aerosp. Eng.* **2014**, 439278.
[28] Von Rueden L et al. 2023. Informed machine learning—a taxonomy and survey of integrating prior knowledge into learning systems. *IEEE Trans. Knowl. Data Eng.* **35**, 614–633.
[29] Schwenke H, Knapp W, Haitjema H, Weckenmann A, Schmitt R and Delbressine F. 2008. Geometric error measurement and compensation of machines-An update. *CIRP Ann.* **57**, 660–675.
[30] Li Y, Zhao W H, Lan S H, Ni J, Wu W W and Lu B H. 2015. A review on spindle thermal error compensation in machine tools. *Int. J. Mach. Tools Manuf.* **95**, 20–38.
[31] Lyu D, Liu Q, Liu H and Zhao W H. 2020. Dynamic error of CNC machine tools: a state-of-the-art review. *Int. J. Adv. Manuf. Technol.* **106**, 1869–1891.
[32] Aheleroff S, Xu X, Zhong R Y and Lu Y Q. 2021. Digital twin as a service (DTaaS) in industry 4.0: an architecture reference model. *Adv. Eng. Inf.* **47**, 101225.
[33] Fuller A, Fan Z, Day C and Barlow C. 2020. Digital twin: enabling technologies, challenges and open research. *IEEE Access* **8**, 108952–108971.
[34] Ji S, Ni H P, Hu T L, Sun J, Yu H W and Jin H Z. 2024. DT-CEPA: a digital twin-driven contour error prediction approach for machine tools based on hybrid modeling and sparse time series. *Robot. Comput.-Integr. Manuf.* **88**, 102738.
[35] Rosado J, Aristimuño P X and Arrazola P J. 2023. Digital twin for final generated surface dimensional error analysis at tool path level in contour milling. *Proc. CIRP* **117**, 44–49.
[36] Li R Y, Zhao L B, Zhou B and Zhao W K. 2023. Research on machining error control method driven by digital-twin model of dynamic characteristics of machining system. *Integr. Ferroelectr.* **237**, 321–335.
[37] Ma C, Gui H Q and Liu J L. 2023. Self learning-empowered thermal error control method of precision machine tools based on digital twin. *J. Intell. Manuf.* **34**, 695–717.
[38] Liu J L, Ma C and Yuan Q. 2024. Spindle unit thermal error modeling and compensation based on digital twin. *Int. J. Adv. Manuf. Technol.* **132**, 1525–1555.
[39] Liu J L, Ma C, Gui H Q and Wang S L. 2023. Intelligent digital-twin prediction and reverse control system architecture for thermal errors enabled by deep learning and cloud-edge computing. *Expert Syst. Appl.* **225**, 120122.
[40] Ma S, Leng J W, Zheng P, Chen Z Y, Li B, Li W H, Liu Q and Chen X. 2025. A digital twin-assisted deep transfer learning method towards intelligent thermal error modeling of electric spindles. *J. Intell. Manuf.* **36**, 1659–1688.
[41] Liu K, Song L, Han W, Cui Y M and Wang Y Q. 2022. Time-varying error prediction and compensation for movement axis of CNC machine tool based on digital twin. *IEEE Trans. Ind. Inf.* **18**, 109–118.

[42] Li R Y, Wang S C, Wang C, Wang S S, Zhou B, Liu X L and Zhao X D. 2023. Research into dynamic error optimization method of impeller blade machining based on digital-twin technology. *Machines* **11**, 697.
[43] Lyu D, Liu J, Luo S Y, Liu S, Cheng Q L and Liu H. 2022. Digital twin modelling method of five-axis machine tool for predicting continuous trajectory contour error. *Processes* **10**, 2725.
[44] Abed M, Gameros A, Mohammad A and Axinte D. 2023. Swift feedback and immediate error control using a lightweight simulation approach—a case study of the digital-twin-in-the-loop for machining thin-wall structures. *J. Manuf. Syst.* **71**, 309–322.
[45] Liu J L, Gui H Q and Ma C. 2021. Digital twin system of thermal error control for a large-size gear profile grinder enabled by gated recurrent unit. *J. Ambient Intell. Human Comput.* **14**, 1269–1295.
[46] Chen J L, An Q L, Chen M, Ko T J and Chen T. 2025. Digital twin dynamic force-thermal physics sub-cell for CFRP drilling process. *Composites B* **292**, 112057.
[47] Irino N, Kobayashi A, Shinba Y, Kawai K, Spescha D and Wegener K. 2023. Digital twin based accuracy compensation. *CIRP Ann.* **72**, 345–348.
[48] Zheng F Y, Han X H, Hua L and Xu W J. 2025. Digital twin of dynamics for parallel kinematic machine with distributed force/position interaction. *J. Manuf. Syst.* **80**, 70–88.
[49] Sa G, Sun J C, Hou M J, Jiang Z Y, Liu Z Y, Mao H Y, Huang K, He L and Tan J R. 2024. A digital twin synchronous evolution method of CNC machine tools associated with dynamic and static errors. *Int. J. Adv. Manuf. Technol.* **134**, 2753–2763.
[50] Gao W, Ibaraki S, Donmez M A, Kono D, Mayer J R R, Chen Y L, Szipka K, Archenti A, Linares J M and Suzuki N. 2023. Machine tool calibration: measurement, modeling, and compensation of machine tool errors. *Int. J. Mach. Tools Manuf.* **187**, 104017.
[51] Wang Y, Cao Y, Qu X R, Wang M, Wang Y L and Zhang C. 2025. A review of the application of machine learning techniques in thermal error compensation for CNC machine tools. *Measurement* **243**, 116341.
[52] Feiyan G, Jianhua L, Fang Z, Yunong Z, Zhongqi W and Shaozhuo S Z. 2019. Research on the state-of-art, connotation and key implementation technology of assembly process planning with digital twin. *J. Mech. Eng.* **55**, 110–132.
[53] Jiang Z M, Guo Y M and Wang Z Q. 2021. Digital twin to improve the virtual-real integration of industrial IoT. *J. Ind. Inf. Integr.* **22**, 100196.
[54] Isah A, Shin H, Oh S, Oh S, Aliyu I, Um T W and Kim J. 2023. Digital twins temporal dependencies-based on time series using multivariate long short-term memory. *Electronics* **12**, 4187.
[55] Mayer J R R. 2012. Five-axis machine tool calibration by probing a scale enriched reconfigurable uncalibrated master balls artefact. *CIRP Ann.* **61**, 515–518.
[56] Rahman M M and Mayer J R R. 2015. Five axis machine tool volumetric error prediction through an indirect estimation of intra- and inter-axis error parameters by probing facets on a scale enriched uncalibrated indigenous artefact. *Precis. Eng.* **40**, 94–105.
[57] Biermann D, Holtermann R, Menzel A and Schumann S. 2016. Modelling and simulation of thermal effects in internal traverse grinding of hardened bearing steel. *CIRP Ann.* **65**, 321–324.
[58] ISO. 2014. ISO 230-2:2014 Test code for machine tools–part 2: determination of accuracy and repeatability of positioning of numerically controlled axes. (ISO)
[59] Bringmann B and Knapp W. 2009. Machine tool calibration: geometric test uncertainty depends on machine tool performance. *Precis. Eng.* **33**, 524–529.
[60] Gebhardt M, Mayr J, Furrer N, Widmer T, Weikert S and Knapp W. 2014. High precision grey-box model for compensation of thermal errors on five-axis machines. *CIRP Ann.* **63**, 509–512.
[61] Stoop F, Mayr J, Sulz C, Kaftan P, Bleicher F, Yamazaki K and Wegener K. 2023. Cloud-based thermal error compensation with a federated learning approach. *Precis. Eng.* **79**, 135–145.
[62] Kono D, Matsubara A, Nagaoka K and Yamazaki K. 2012. Analysis method for investigating the influence of mechanical components on dynamic mechanical error of machine tools. *Precis. Eng.* **36**, 477–484.
[63] Xi X C, Poo A N and Hong G S. 2009. Improving contouring accuracy by tuning gains for a bi-axial CNC machine. *Int. J. Mach. Tools Manuf.* **49**, 395–406.
[64] Ibaraki S, Oyama C and Otsubo H. 2011. Construction of an error map of rotary axes on a five-axis machining center by static R-test. *Int. J. Mach. Tools Manuf.* **51**, 190–200.
[65] Eynian M and Altintas Y. 2009. Chatter stability of general turning operations with process damping. *J. Manuf. Sci. Eng.* **131**, 041005.
[66] Sun Y W and Jiang S L. 2018. Predictive modeling of chatter stability considering force-induced deformation effect in milling thin-walled parts. *Int. J. Mach. Tools Manuf.* **135**, 38–52.
[67] Ding S, Huang X D, Yu C J and Wang W. 2016. Actual inverse kinematics for position-independent and position-dependent geometric error compensation of five-axis machine tools. *Int. J. Mach. Tools Manuf.* **111**, 55–62.
[68] Möhring H C, Wiederkehr P, Erkorkmaz K and Kakinuma Y. 2020. Self-optimizing machining systems. *CIRP Ann.* **69**, 740–763
[69] Gao W, Chen Y L, Lee K W, Noh Y J, Shimizu Y and Ito S. 2013. Precision tool setting for fabrication of a microstructure array. *CIRP Ann.* **62**, 523–526.
[70] Zimmermann N, Lang S, Blaser P and Mayr J. 2020. Adaptive input selection for thermal error compensation models. *CIRP Ann.* **69**, 485–488.
[71] Esmaeili S M and Mayer J R R. 2021. CNC table based compensation of inter-axis and linear axis scale gain errors for a five-axis machine tool from symbolic variational kinematics. *CIRP Ann.* **70**, 439–442.
[72] Mayr J et al. 2012. Thermal issues in machine tools. *CIRP Ann.* **61**, 771–791.
[73] Bryan J. 1990. International status of thermal error research. *CIRP Ann.* **39**, 645–656.
[74] Lei M, Gao F, Li Y, Xia P, Wang M C and Yang J. 2022. Feedback control-based active cooling with pre-estimated reliability for stabilizing the thermal error of a precision mechanical spindle. *Int. J. Adv. Manuf. Technol.* **121**, 2023–2040.
[75] Liu T, Li C T, Zhang Y F, Gao W G, Fu Z K, Zhang J J and Zhang D W. 2022. Active coolant control onto thermal behaviors of precision ball screw unit. *Int. J. Adv. Manuf. Technol.* **119**, 1867–1882.
[76] Zhang C X, Gao F and Li Y. 2017. Thermal error characteristic analysis and modeling for machine tools due to time-varying environmental temperature. *Precis. Eng.* **47**, 231–238.
[77] Tan B, Mao X Y, Liu H Q, Li B, He S P, Peng F Y and Yin L. 2014. A thermal error model for large machine tools that considers environmental thermal hysteresis effects. *Int. J. Mach. Tools Manuf.* **82-83**, 11–20.
[78] Weng L T, Gao W G, Lv Z S, Zhang D W, Liu T, Wang Y, Qi X Y and Tian Y L. 2018. Influence of external heat sources on volumetric thermal errors of precision machine tools. *Int. J. Adv. Manuf. Technol.* **99**, 475–495.
[79] Ge Z and Ding X H. 2018. Thermal error control method based on thermal deformation balance principle for the precision

parts of machine tools. *Int. J. Adv. Manuf. Technol.* **97**, 1253–1268.
[80] Namaba Y, Wada R, Unno K, Tsuboi A and Okamura K. 1989. Ultra-precision surface grinder having a glass-ceramic spindle of zero-thermal expansion. *CIRP Ann.* **38**, 331–334.
[81] Putz M, Richter C, Regel J and Bräunig M. 2018. Industrial consideration of thermal issues in machine tools. *Prod. Eng.* **12**, 723–736.
[82] Wegener K et al. 2017. Fluid elements in machine tools. *CIRP Ann.* **66**, 611–634.
[83] Grama S N, Mathur A and Badhe A N. 2018. A model-based cooling strategy for motorized spindle to reduce thermal errors. *Int. J. Mach. Tools Manuf.* **132**, 3–16.
[84] Chang C F and Chen J J. 2009. Thermal growth control techniques for motorized spindles. *Mechatronics* **19**, 1313–1320.
[85] Weber J, Shabi L and Weber J. 2018. State of the art and optimization of the energy flow in cooling systems of motorized high-speed spindles in machine tools *Proc. CIRP* **67**, 81–86.
[86] Xia C H, Fu J Z, Lai J T, Yao X H and Chen Z C. 2015. Conjugate heat transfer in fractal tree-like channels network heat sink for high-speed motorized spindle cooling. *Appl. Therm. Eng.* **90**, 1032–1042.
[87] Xu Z Z, Liu X J, Kim H K, Shin J H and Lyu S K. 2011. Thermal error forecast and performance evaluation for an air-cooling ball screw system. *Int. J. Mach. Tools Manuf.* **51**, 605–611.
[88] Xu Z Z, Liu X J and Lyu S K. 2014. Study on thermal behavior analysis of nut/shaft air cooling ball screw for high-precision feed drive. *Int. J. Precis. Eng. Manuf.* **15**, 123–128.
[89] Shi H, He B, Yue Y Y, Min C Q and Mei X S. 2019. Cooling effect and temperature regulation of oil cooling system for ball screw feed drive system of precision machine tool. *Appl. Therm. Eng.* **161**, 114150.
[90] Huang S, Liu S, Wang D Z, Takao A, Wu S J, Li C, Xiang D H and Li C H. 2024. Bionic design and optimization of cutting tools: applications and processability. *J. Manuf. Process.* **131**, 1086–1131.
[91] Huang S, Liu S, Wang D Z, Wu S J, Wang G Q, Wan L, An Q L, Zhu L D and Li C H. 2025. Bio-inspired cutting tools: beneficial mechanisms, fabrication technology and coupling design. *Sustain. Mater. Technol.* **43**, e01211.
[92] Huang W T, Wu D H and Chen J T. 2016. Robust design of using nanofluid/MQL in micro-drilling. *Int. J. Adv. Manuf. Technol.* **85**, 2155–2161.
[93] Zhang Y B, Li C H, Jia D Z, Li B K, Wang Y G, Yang M, Hou Y L, Zhang N Q and Wu Q D. 2018. Experimental evaluation of the workpiece surface quality of MoS_2/CNT nanofluid for minimal quantity lubrication in grinding. *J. Mech. Eng.* **54**, 161–170.
[94] Tong J W, Peng R T and Song Q H. 2023. A Cu-based ceramic porous abrasive ring with phyllotactic pattern in internal cooling grinding of Inconel 718. *J. Mater. Process. Technol.* **315**, 117890.
[95] Peng R T, Zhao L F, Tong J W, Chen M L, Zhou M Z and Li A. 2022. Design and evaluation of an internal-cooling grooved grinding wheel. *J. Manuf. Process.* **73**, 1–16.
[96] O'Hara J and Fang F Z. 2019. Advances in micro cutting tool design and fabrication. *Int. J. Extrem. Manuf.* **1**, 032003.
[97] Bleicher F and Reiter M. 2018. Wear reduction on cutting inserts by additional internal cooling of the cutting edge. *Proc. Manuf.* **21**, 518–524.
[98] Liu H Z, Yan Y D, Cui J W, Geng Y Q, Sun T, Luo X C and Zong W J. 2024. Recent advances in design and preparation of micro diamond cutting tools. *Int. J. Extrem. Manuf.* **6**, 062008.
[99] Ibaraki S and Okumura R. 2021. A machining test to evaluate thermal influence on the kinematics of a five-axis machine tool. *Int. J. Mach. Tools Manuf.* **163**, 103702.
[100] Huang Y B, Fan K C, Lou Z F and Sun W. 2020. A novel modeling of volumetric errors of three-axis machine tools based on Abbe and Bryan principles. *Int. J. Mach. Tools Manuf.* **151**, 103527.
[101] Liu T, Gao W G, Tian Y L, Zhang H J, Chang W F, Mao K and Zhang D W. 2015. A differentiated multi-loops bath recirculation system for precision machine tools. *Appl. Therm. Eng.* **76**, 54–63.
[102] Creighton E, Honegger A, Tulsian A and Mukhopadhyay D. 2010. Analysis of thermal errors in a high-speed micro-milling spindle. *Int. J. Mach. Tools Manuf.* **50**, 386–393.
[103] Oyanguren A, Larrañaga J and Ulacia I. 2018. Thermo-mechanical modelling of ball screw preload force variation in different working conditions. *Int. J. Adv. Manuf. Technol.* **97**, 723–739.
[104] Liu J L, Ma C, Wang S L, Wang S B, Yang B and Shi H. 2019. Thermal-structure interaction characteristics of a high-speed spindle-bearing system. *Int. J. Mach. Tools Manuf.* **137**, 42–57.
[105] Liu T, Gao W G, Tian Y L, Zhang D W, Zhang Y F and Chang W F. 2017. Power matching based dissipation strategy onto spindle heat generations. *Appl. Therm. Eng.* **113**, 499–507.
[106] Shi H, Ma C, Yang J, Zhao L, Mei X S and Gong G F. 2015. Investigation into effect of thermal expansion on thermally induced error of ball screw feed drive system of precision machine tools. *Int. J. Mach. Tools Manuf.* **97**, 60–71.
[107] Yao X P, Hu T, Yin G F and Cheng C H. 2020. Thermal error modeling and prediction analysis based on OM algorithm for machine tool's spindle. *Int. J. Adv. Manuf. Technol.* **106**, 3345–3356.
[108] Zhang J F, Feng P F, Chen C, Yu D W and Wu Z J. 2013. A method for thermal performance modeling and simulation of machine tools. *Int. J. Adv. Manuf. Technol.* **68**, 1517–1527.
[109] Sa G D, Jiang Z Y, Liu Z Y, Sun J C, Qiu C, He L and Tan J R. 2024. An integrated optimization method for measurement points layout and error modeling for digital twin of CNC machine tools. *Precis. Eng.* **90**, 1–11.
[110] Liu Y C, Liao C W and Lee M T. 2024. Active thermal balance control for improving orientation error of vertical machine tool spindle. *Precis. Eng.* **88**, 686–698.
[111] Zeng S, Ma C, Liu J L, Li M Y and Gui H Q. 2023. Sequence-to-sequence based LSTM network modeling and its application in thermal error control framework. *Appl. Soft Comput.* **138**, 110221.
[112] Tounsi N and Otho A. 2000. Identification of machine–tool–workpiece system dynamics. *Int. J. Mach. Tools Manuf.* **40**, 1367–1384.
[113] Kong X G, Li C K, Li Z H, Yang M, Cui X, Liu M Z, Li B K, Zhang Y B, Ma X and Li C H. 2025. Biological bone and replacement materials in grinding: force model and processing capability. *Intell. Sustain. Manuf.* **2**, 10003.
[114] Sun J G et al. 2023. Material removal mechanism and force modeling in ultrasonic vibration-assisted micro-grinding biological bone. *Chin. J. Mech. Eng.* **36**, 129.
[115] An Q L, Yang J, Li J L, Liu G, Chen M and Li C H. 2024. A state-of-the-art review on the intelligent tool holders in machining. *Intell. Sustain. Manuf.* **1**, 10002
[116] Xin L J, Zhao G L, Nian Z W, Yang H T, Li L and He N. 2025. Physics-based modeling and mechanism of polycrystalline diamond tool wear in milling of 70 vol% Si/Al composite. *Int. J. Extrem. Manuf.* **7**, 055102.
[117] Weng L T, Gao W G, Zhang D W, Huang T, Liu T, Li W, Zheng Y J, Shi K and Chang W F. 2021. Analytical modelling method for thermal balancing design of machine

tool structural components. *Int. J. Mach. Tools Manuf.* **164**, 103715.
[118] Sun H, Zhao S Q, Peng F Y, Yan R, Zhou L, Zhang T and Zhang C. 2024. In-situ prediction of machining errors of thin-walled parts: an engineering knowledge based sparse Bayesian learning approach. *J. Intell. Manuf.* **35**, 387–411.
[119] Suzuki N, Takahashi W, Igeta H and Nakanomiya T. 2020. Flank face texture design to suppress chatter vibration in cutting. *CIRP Ann.* **69**, 93–96.
[120] Takahashi W, Nakanomiya T, Suzuki N and Shamoto E. 2021. Influence of flank texture patterns on the suppression of chatter vibration and flank adhesion in turning operations. *Precis. Eng. Nanotechnol.* **68**, 262–272.
[121] Kono D, Matsubara A, Shirai T, Hoshide K, Miura T and Togashi T. 2016. Analysis of positional deviation caused by position-dependent disturbances in ball screw drive. *J. Japan Soc. Precis. Eng.* **82**, 589–594.
[122] Suzuki N, Ishiguro R and Kojima T. 2016. Design of irregular pitch end mills to attain robust suppression of regenerative chatter. *CIRP Ann.* **65**, 129–132.
[123] Nam S, Hayasaka T, Jung H J and Shamoto E. 2021. Proposal of novel spindle speed variation profile with constant acceleration rate for improvement of chatter stability. *Precis. Eng.* **68**, 218–234.
[124] Li D H, Cao H R, Liu J X, Zhang X W and Chen X F. 2019. Milling chatter control based on asymmetric stiffness. *Int. J. Mach. Tools Manuf.* **147**, 103458.
[125] Gubanov G. 2020. Broadband pneumatic mass damper for the elimination of workpiece vibrations. *CIRP J. Manuf. Sci. Technol.* **30**, 184–194.
[126] Ding J G, Chang Y, Chen P, Zhuang H, Ding Y Y, Lu H J and Chen Y H. 2020. Dynamic modeling of ultra-precision fly cutting machine tool and the effect of ambient vibration on its tool tip response. *Int. J. Extrem. Manuf.* **2**, 025301.
[127] Zhao B et al. 2024. Overcoming challenges: advancements in cutting techniques for high strength-toughness alloys in aero-engines. *Int. J. Extrem. Manuf.* **6**, 062012.
[128] Ramesh R, Mannan M A and Poo A N. 2005. Tracking and contour error control in CNC servo systems. *Int. J. Mach. Tools Manuf.* **45**, 301–326.
[129] Kong X Y, Wang L P, Yu G, Li W T and Li M Y. 2023. Research on servo matching of a five-axis hybrid machine tool. *Int. J. Adv. Manuf. Technol.* **129**, 983–997.
[130] Guan L W, Chen Y Y and Wang Z J. 2025. Dynamic performance optimization of five-axis machine tools based on S-specimen and servo dynamics matching degree indexes. *Int. J. Adv. Manuf. Technol.* **137**, 3887–3913.
[131] Liu W J, Zhang S, Lin J H, Xia Y H, Wang J X and Sun Y L. 2022. Advancements in accuracy decline mechanisms and accuracy retention approaches of CNC machine tools: a review. *Int. J. Adv. Manuf. Technol.* **121**, 7087–7115.
[132] Chen Y L, Cai Y D, Shimizu Y, Ito S, Gao W and Ju B F. 2016. On-machine measurement of microtool wear and cutting edge chipping by using a diamond edge artifact. *Precis. Eng.* **43**, 462–467.
[133] Fan J W, Wang X F and Li Y. 2012. Research on analytical method of CNC machine tool precision recession. *Appl. Mech. Mater.* **159**, 213–217.
[134] Huang J, Zhou Z D, Liu M Y, Zhang E L, Chen M, Pham D T and Ji C Q. 2015. Real-time measurement of temperature field in heavy-duty machine tools using fiber Bragg grating sensors and analysis of thermal shift errors. *Mechatronics* **31**, 16–21.
[135] Guo S J, Pang B, Jiang G J, Chen H X and Li Z Q. 2020. Time-varying reliability modelling and quasi-static accuracy optimization of precision CNC machine tools. *J. Phys.: Conf. Ser.* **1654**, 012022.
[136] Wang Y Q, Wu J K, Liu H B, Kang K and Liu K. 2018. Geometric accuracy long-term continuous monitoring using strain gauges for CNC machine tools. *Int. J. Adv. Manuf. Technol.* **98**, 1145–1153.
[137] Lee W J, Wu H, Yun H, Kim H, Jun M B G and Sutherland J W. 2019. Predictive maintenance of machine tool systems using artificial intelligence techniques applied to machine condition data. *Proc. CIRP* **80**, 506–511.
[138] Fujishima M, Narimatsu K, Irino N, Mori M and Ibaraki S. 2019. Adaptive thermal displacement compensation method based on deep learning. *CIRP J. Manuf. Sci. Technol.* **25**, 22–25.
[139] Wang L P, Kong X Y, Yu G, Li W T, Li M Y and Jiang A B. 2022. Error estimation and cross-coupled control based on a novel tool pose representation method of a five-axis hybrid machine tool. *Int. J. Mach. Tools Manuf.* **182**, 103955.
[140] Wang T X, Shi J, Sun J H and Zhang S. 2025. Position error decomposition and prediction of CNC machine tool under thermal-mechanical coupling loads. *Int. J. Adv. Manuf. Technol.* **137**, 199–216.
[141] Zhao Z W, Mao J and Wei X C. 2024. Geometric error-based multi-source error identification and compensation strategy for five-axis side milling. *Machines* **12**, 340.
[142] Han Y, Deng X L, Zheng J J, Lin X L, Wang X Y and Chen Y. 2024. Thermal error prediction for vertical machining centers using decision-level fusion of multi-source heterogeneous information. *Machines* **12**, 509.
[143] Szipka K, Archenti A, Vogl G W and Donmez M A. 2019. Identification of machine tool squareness errors via inertial measurements. *CIRP Ann.* **68**, 547–550.
[144] Zhao G L, Zhao B, Ding W F, Xin L J, Nian Z W, Peng J H, He N and Xu J H. 2024. Nontraditional energy-assisted mechanical machining of difficult-to-cut materials and components in aerospace community: a comparative analysis. *Int. J. Extrem. Manuf.* **6**, 022007.
[145] Liu C, Xiang S T, Lu C W, Wu C Y, Du Z C and Yang J G. 2020. Dynamic and static error identification and separation method for three-axis CNC machine tools based on feature workpiece cutting. *Int. J. Adv. Manuf. Technol.* **107**, 2227–2238.
[146] Aguado S, Santolaria J, Samper D, Aguilar J J and Velázquez J. 2016. Improving a real milling machine accuracy through an indirect measurement of its geometric errors. *J. Manuf. Syst.* **40**, 26–36.
[147] ISO. 2012. ISO 230-1:2012 Test code for machine tools–part 1: geometric accuracy of machines operating under no-load or quasi-static conditions. (ISO)
[148] Gao W, Saito Y, Muto H, Arai Y and Shimizu Y. 2011. A three-axis autocollimator for detection of angular error motions of a precision stage. *CIRP Ann.* **60**, 515–518.
[149] Liu C S, Hsu H C and Lin Y X. 2020. Design of a six-degree-of-freedom geometric errors measurement system for a rotary axis of a machine tool. *Opt. Lasers Eng.* **127**, 105949.
[150] Kumar A S, Agarwal A, Jansari V G, Desai K A, Chattopadhyay C and Mears L. 2025. Realizing on-machine tool wear monitoring through integration of vision-based system with CNC milling machine. *J. Manuf. Syst.* **78**, 283–293.
[151] Chen B Y, Zhang E Z, Yah L P, Li C R, Tang W H and Feng Q B. 2009. A laser interferometer for measuring straightness and its position based on heterodyne interferometry. *Rev. Sci. Instrum.* **80**, 115113.
[152] Lee D M, Lee H H and Yang S H. 2013. Analysis of squareness measurement using a laser interferometer system. *Int. J. Precis. Eng. Manuf.* **14**, 1839–1846.

[153] Wang W, Kweon S H, Hwang C S, Kang N C, Kim Y S and Yang S H. 2009. Development of an optical measuring system for integrated geometric errors of a three-axis miniaturized machine tool. *Int. J. Adv. Manuf. Technol.* **43**, 701–709.
[154] Nozato H, Kokuyama W and Ota A. 2016. Improvement and validity of shock measurements using heterodyne laser interferometer. *Measurement* **77**, 67–72.
[155] Hsieh H L, Chen J C, Lerondel G and Lee J Y. 2011. Two-dimensional displacement measurement by quasi-common-optical-path heterodyne grating interferometer. *Opt. Express* **19**, 9770–9782.
[156] Lu Y C et al. 2016. Two-degree-freedom displacement measurement based on a short period grating in symmetric Littrow configuration. *Opt. Commun.* **380**, 382–386.
[157] Shimizu Y, Tan S L, Murata D, Maruyama T, Ito S, Chen Y L and Gao W. 2016. Ultra-sensitive angle sensor based on laser autocollimation for measurement of stage tilt motions. *Opt. Express* **24**, 2788–2805.
[158] Shimizu Y, Kataoka S, Ishikawa T, Chen Y L, Chen X G, Matsukuma H and Gao W. 2018. A liquid-surface-based three-axis inclination sensor for measurement of stage tilt motions. *Sensors* **18**, 398.
[159] Chen C J, Lin P D and Jywe W Y. 2007. An optoelectronic measurement system for measuring 6-degree-of-freedom motion error of rotary parts. *Opt. Express* **15**, 14601–14617.
[160] Kono D and Kondo S. 2024. Accuracy evaluation of squareness identification by vision-based circular tests for machine tools. *CIRP Ann.* **73**, 309–312.
[161] Huang Z H et al. 2025. Mechanisms of machine vision feature recognition and quality prediction models in intelligent production line for broiler carcasses. *Intell. Sustain. Manuf.* **2**, 10016.
[162] Li X, Liu W, Pan Y, Liang B, Zhou M D, Li H, Wang F J and Jia Z Y. 2019. Monocular-vision-based contouring error detection and compensation for CNC machine tools. *Precis. Eng.* **55**, 447–463.
[163] Makhesana M A, Bagga P J, Patel K M, Patel H D, Balu A and Khanna N. 2024. Comparative analysis of different machine vision algorithms for tool wear measurement during machining. *J. Intell. Manuf.* **36**, 1–25.
[164] Ibaraki S and Hata T. 2010. A new formulation of laser step diagonal measurement-Three-dimensional case. *Precis. Eng.* **34**, 516–525.
[165] Hong C F, Ibaraki S and Oyama C. 2012. Graphical presentation of error motions of rotary axes on a five-axis machine tool by static R-test with separating the influence of squareness errors of linear axes. *Int. J. Mach. Tools Manuf.* **59**, 24–33.
[166] ISO. 2002. ISO 230-6:2002 Test code for machine tools–part 6: determination of positioning accuracy on body and face diagonals (diagonal displacement tests). (Geneva, Switzerland: ISO)
[167] Wang C. 2000. Laser vector measurement technique for the determination and compensation of volumetric positioning errors. Part I: basic theory. *Rev. Sci. Instrum.* **71**, 3933–3937.
[168] Zhang H T, Yang J G, Zhang Y, Shen J H and Wang C. 2011. Measurement and compensation for volumetric positioning errors of CNC machine tools considering thermal effect. *Int. J. Adv. Manuf. Technol.* **55**, 275–283.
[169] Bui C B, Hwang J, Lee C H and Park C H. 2012. Three-face step-diagonal measurement method for the estimation of volumetric positioning errors in a 3D workspace. *Int. J. Mach. Tools Manuf.* **60**, 40–43.
[170] Li J, Xie F G, Liu X J, Li W D and Zhu S W. 2016. Geometric error identification and compensation of linear axes based on a novel 13-line method. *Int. J. Adv. Manuf. Technol.* **87**, 2269–2283.
[171] Huang Y B, Fan K C, Sun W and Liu S J. 2018. Low cost, compact 4-DOF measurement system with active compensation of beam angular drift error. *Opt. Express* **26**, 17185–17198.
[172] Sun C, Cai S, Liu Y S and Qiao Y F. 2020. Compact laser collimation system for simultaneous measurement of five-degree-of-freedom motion errors. *Appl. Sci.* **10**, 5057.
[173] Aguado S, Santolaria J, Samper D and Aguilar J J. 2013. Influence of measurement noise and laser arrangement on measurement uncertainty of laser tracker multilateration in machine tool volumetric verification. *Precis. Eng.* **37**, 929–943.
[174] Wang J D and Guo J J. 2016. Geometric error identification algorithm of numerical control machine tool using a laser tracker. *Proc. Inst. Mech. Eng. B* **230**, 2004–2015.
[175] Xia H J, Peng W C, Ouyang X B, Chen X D, Wang S J and Chen X. 2017. Identification of geometric errors of rotary axis on multi-axis machine tool based on kinematic analysis method using double ball bar. *Int. J. Mach. Tools Manuf.* **122**, 161–175.
[176] Zhang Y, Fu J Z and Chen Z C. 2013. Machining tests to identify kinematic errors of machine tool table rotation axis based on sensitive directions. *Int. J. Adv. Manuf. Technol.* **67**, 495–500.
[177] Tian W J, Yang G, Wang L N, Yin F W and Gao W G. 2018. The application of a regularization method to the estimation of geometric errors of a three-axis machine tool using a double ball bar. *J. Mech. Sci. Technol.* **32**, 4871–4881.
[178] Tsutsumi M and Saito A. 2004. Identification of angular and positional deviations inherent to 5-axis machining centers with a tilting-rotary table by simultaneous four-axis control movements. *Int. J. Mach. Tools Manuf.* **44**, 1333–1342.
[179] Jiang X G and Cripps R J. 2016. Geometric characterisation and simulation of position independent geometric errors of five-axis machine tools using a double ball bar. *Int. J. Adv. Manuf. Technol.* **83**, 1905–1915.
[180] Archenti A, Nicolescu M, Casterman G and Hjelm S. 2012. A new method for circular testing of machine tools under loaded condition. *Proc. CIRP* **1**, 575–580.
[181] Szipka K, Laspas T and Archenti A. 2018. Measurement and analysis of machine tool errors under quasi-static and loaded conditions. *Precis. Eng.* **51**, 59–67.
[182] Jiang X G and Cripps R J. 2015. A method of testing position independent geometric errors in rotary axes of a five-axis machine tool using a double ball bar. *Int. J. Mach. Tools Manuf.* **89**, 151–158.
[183] Xiang S T and Yang J G. 2014. Using a double ball bar to measure 10 position-dependent geometric errors for rotary axes on five-axis machine tools. *Int. J. Adv. Manuf. Technol.* **75**, 559–572.
[184] Weikert S. 2004. R-test, a new device for accuracy measurements on five axis machine tools. *CIRP Ann.* **53**, 429–432.
[185] Liebrich T, Bringmann B and Knapp W. 2009. Calibration of a 3D-ball plate. *Precis. Eng.* **33**, 1–6.
[186] Lei W T and Hsu Y Y. 2002. Accuracy test of five-axis CNC machine tool with 3D probe–ball. Part I: design and modeling. *Int. J. Mach. Tools Manuf.* **42**, 1153–1162.
[187] Lei W T and Hsu Y Y. 2002. Accuracy test of five-axis CNC machine tool with 3D probe–ball. Part II: errors estimation. *Int. J. Mach. Tools Manuf.* **42**, 1163–1170.
[188] Erkan T and Mayer J R R. 2010. A cluster analysis applied to volumetric errors of five-axis machine tools obtained by probing an uncalibrated artefact. *CIRP Ann.* **59**, 539–542.
[189] Lei W T, Sung M P, Liu W L and Chuang Y C. 2007. Double ballbar test for the rotary axes of five-axis CNC machine tools. *Int. J. Mach. Tools Manuf.* **47**, 273–285.
[190] Li M Y, Wang L P, Yu G, Li W T and Kong X Y. 2023. A multiple test arbors-based calibration method for a hybrid machine tool. *Robot. Comput.-Integr. Manuf.* **80**, 102480.

[191] Lee J H and Yang S H. 2002. Statistical optimization and assessment of a thermal error model for CNC machine tools. *Int. J. Mach. Tools Manuf.* **42**, 147–155.
[192] Tootooni M S, Liu C A, Roberson D, Donovan R, Rao P K, Kong Z Y and Bukkapatnam S T S. 2016. Online non-contact surface finish measurement in machining using graph theory-based image analysis. *J. Manuf. Syst.* **41**, 266–276.
[193] Wang Q Y, Jiao W H and Zhang Y M. 2020. Deep learning-empowered digital twin for visualized weld joint growth monitoring and penetration control. *J. Manuf. Syst.* **57**, 429–439.
[194] Dambatta Y S et al. 2025. Digital twin and artificial intelligence in machining: a bibliometric analysis. *Intell. Sustain. Manuf.* **2**, 10005.
[195] Botín-Sanabria D M, Mihaita A S, Peimbert-García R E, Ramírez-Moreno M A, Ramírez-Mendoza R A and Lozoya-Santos J D. 2022. Digital twin technology challenges and applications: a comprehensive review. *Remote Sens.* **14**, 1335.
[196] Hu S G, Li C H, Li B K, Yang M, Wang X M, Gao T, Xu W H, Dambatta Y S, Zhou Z M and Xu P M. 2024. Digital twins enabling intelligent manufacturing: from methodology to application. *Intell. Sustain. Manuf.* **1**, 10007.
[197] Mohammadi A, Züst S, Mayr J, Blaser P, Sonne M R, Hattel J H and Wegener K. 2017. A methodology for online visualization of the energy flow in a machine tool. *CIRP J. Manuf. Sci. Technol.* **19**, 138–146.
[198] Vyroubal J. 2012. Compensation of machine tool thermal deformation in spindle axis direction based on decomposition method. *Precis. Eng.* **36**, 121–127.
[199] Qin Y, Wu X G and Luo J. 2022. Data-model combined driven digital twin of life-cycle rolling bearing. *IEEE Trans. Ind. Inf.* **18**, 1530–1540.
[200] Yang Y, Lv J, Xiao Y K, Feng X B and Du Z C. 2025. Enhanced modeling method of thermal behaviors in machine tool motorized spindles based on the mixture of thermal mechanism and machine learning. *J. Intell. Manuf.* **36**, 221–242.
[201] Uhlemann T H J, Schock C, Lehmann C, Freiberger S and Steinhilper R. 2017. The digital twin: demonstrating the potential of real time data acquisition in production systems. *Proc. Manuf.* **9**, 113–120.
[202] Madni A M, Madni C C and Lucero S D. 2019. Leveraging digital twin technology in model-based systems engineering. *Systems* **7**, 7.
[203] Rasheed A, San O and Kvamsdal T. 2020. Digital twin: values, challenges and enablers from a modeling perspective. *IEEE Access* **8**, 21980–22012.
[204] Lu C X, Wang J H, Yin P L and Wang L Y. 2021. Error identification of measurement software based on digital twin of gear measuring center. *Measurement* **173**, 108666.
[205] Qi T F, Fang H R, Chen Y F and He L T. 2024. Research on digital twin monitoring system for large complex surface machining. *J. Intell. Manuf.* **35**, 977–990.
[206] Dai Y Q and Zhu K P. 2018. A machine vision system for micro-milling tool condition monitoring. *Precis. Eng.* **52**, 183–191.
[207] Chen M L, Li M D, Zhao L F and Liu J C. 2023. Tool wear monitoring based on the combination of machine vision and acoustic emission. *Int. J. Adv. Manuf. Technol.* **125**, 3881–3897.
[208] Reddy T N, Shanmugaraj V, Prakash V, Krishna S G, Narendranath S and Kumar P V S. 2014. Real-time thermal error compensation module for intelligent ultra precision turning machine (iUPTM). *Proc. Mater. Sci.* **6**, 1981–1988.
[209] Abdulshahed A M, Longstaff A P, Fletcher S and Potdar A. 2016. Thermal error modelling of a gantry-type 5-axis machine tool using a Grey Neural Network Model. *J. Manuf. Syst.* **41**, 130–142.
[210] Bosetti P and Bruschi S. 2012. Enhancing positioning accuracy of CNC machine tools by means of direct measurement of deformation. *Int. J. Adv. Manuf. Technol.* **58**, 651–662.
[211] Bossmanns B and Tu J F. 1999. A thermal model for high speed motorized spindles. *Int. J. Mach. Tools Manuf.* **39**, 1345–1366.
[212] Ibaraki S and Ota Y. 2014. A machining test to calibrate rotary axis error motions of five-axis machine tools and its application to thermal deformation test. *Int. J. Mach. Tools Manuf.* **86**, 81–88.
[213] Qi Q L, Tao F, Hu T L, Anwer N, Liu A, Wei Y L, Wang L H and Nee A Y C. 2021. Enabling technologies and tools for digital twin. *J. Manuf. Syst.* **58**, 3–21.
[214] Zhao P, Liu J F, Jing X W, Tang M M, Sheng S S, Zhou H G and Liu X J. 2020. The modeling and using strategy for the digital twin in process planning. *IEEE Access* **8**, 41229–41245.
[215] Huang Z Q, Fey M, Liu C, Beysel E, Xu X and Brecher C. 2023. Hybrid learning-based digital twin for manufacturing process: modeling framework and implementation. *Robot. Comput.-Integr. Manuf.* **82**, 102545.
[216] Lynn R, Louhichi W, Parto M, Wescoat E and Kurfess T. 2017. Rapidly deployable MTConnect-based machine tool monitoring systems. *In Proceedings of the 12th International Manufacturing Science and Engineering Conference Collocated with the JSME/ASME 2017 6th International Conference on Materials and Processing* (ASME, Los Angeles, California, USA) p V003T04A046.
[217] Zhang L, Liu J H, Wu X Q and Zhuang C B. 2024. Digital twin-based dynamic prediction of thermomechanical coupling for skiving process. *Int. J. Adv. Manuf. Technol.* **131**, 5471–5488.
[218] Cui Y S, Kara S and Chan K C. 2020. Manufacturing big data ecosystem: a systematic literature review. *Robot. Comput.-Integr. Manuf.* **62**, 101861.
[219] Lee K B, Song E Y and Gu P S. 2012. Integration of MTConnect and standard-based sensor networks for manufacturing equipment monitoring. *In Proceedings of the International Manufacturing Science and Engineering Conference collocated with the 40th North American Manufacturing Research Conference and in participation with the International Conference on Tribology Materials and Processing* (ASME, Notre Dame, Indiana, USA) p 54990.
[220] Li D C and Lin Y S. 2006. Using virtual sample generation to build up management knowledge in the early manufacturing stages. *Eur. J. Oper. Res.* **175**, 413–434.
[221] Patel M, Wang X Y and Mao S W. 2020. Data augmentation with conditional GAN for automatic modulation classification. *In Proceedings of the 2nd ACM Workshop on Wireless Security and Machine Learning* (ACM, Linz Austria) pp 31–36.
[222] Zhang H, Qi Q L, Ji W and Tao F. 2023. An update method for digital twin multi-dimension models. *Robot. Comput.-Integr. Manuf.* **80**, 102481.
[223] Sortino M and Vaglio E. 2024. Automatic classification of smart sensor data for evaluating machine tool efficiency. *Int. J. Adv. Manuf. Technol.* **132**, 6085–6100.
[224] Lang S B S, Zorzini M, Scholze S, Mayr J and Bambach M. 2025. Sensor placement utilizing a digital twin for thermal error compensation of machine tools. *J. Manuf. Syst.* **80**, 243–257.
[225] Zhao F, Zhang C, Yang G L and Chen C Y. 2016. Online machining error estimation method of numerical control gear grinding machine tool based on data analysis of internal sensors. *Mech. Syst. Signal Process.* **81**, 515–526.
[226] Jiang Y C, Drescher B and Yuan G G. 2023. A GAN-based multi-sensor data augmentation technique for CNC

machine tool wear prediction. *IEEE Access*. **11**, 95782–95795.

[227] Kristiani E, Wang L Y, Liu J C, Huang C K, Wei S J and Yang C T. 2024. An intelligent thermal compensation system using edge computing for machine tools. *Sensors* **24**, 2531.

[228] Zhang L, Ma C, Liu J L, Gui H Q and Wang S L. 2023. Implementation of precision machine tool thermal error compensation in edge-cloud-fog computing architecture. *J. Manuf. Sci. Eng., Trans. ASME* **145**, 071004.

[229] Ward R, Sun C, Dominguez-Caballero J, Ojo S, Ayvar-Soberanis S, Curtis D and Ozturk E. 2021. Machining Digital Twin using real-time model-based simulations and lookahead function for closed loop machining control. *Int. J. Adv. Manuf. Technol.* **117**, 3615–3629.

[230] Wu L B et al. 2025. Intelligent manufacturing factory: a bibliometric analysis of global research hotspots and progress. *Intell. Sustain. Manuf.* **1**, 10018.

[231] Majda P. 2012. Modeling of geometric errors of linear guideway and their influence on joint kinematic error in machine tools. *Precis. Eng.* **36**, 369–378.

[232] Yang J X and Altintas Y. 2013. Generalized kinematics of five-axis serial machines with non-singular tool path generation. *Int. J. Mach. Tools Manuf.* **75**, 119–132.

[233] Nguyen V H, Le T T, Truong H S, Duong H T and Le M V. 2023. Predicting volumetric error compensation for five-axis machine tool using machine learning. *Int. J. Comput. Integr. Manuf.* **36**, 1191–1218.

[234] Okafor A C and Ertekin Y M. 2000. Derivation of machine tool error models and error compensation procedure for three axes vertical machining center using rigid body kinematics. *Int. J. Mach. Tools Manuf.* **40**, 1199–1213.

[235] Yu Z, Tiemin L and Xiaoqiang T. 2011. Geometric error modeling of machine tools based on screw theory. *Proc. Eng.* **24**, 845–849.

[236] Yang J X, Mayer J R R and Altintas Y. 2015. A position independent geometric errors identification and correction method for five-axis serial machines based on screw theory. *Int. J. Mach. Tools Manuf.* **95**, 52–66.

[237] Tian W J, Gao W G, Zhang D W and Huang T. 2014. A general approach for error modeling of machine tools. *Int. J. Mach. Tools Manuf.* **79**, 17–23.

[238] Li Y, Yu M L, Bai Y M, Hou Z Y and Wu W W. 2021. A review of thermal error modeling methods for machine tools. *Appl. Sci.* **11**, 5216.

[239] Wang W, Yang J G, Yao X D, Fan K G and Li Z H. 2012. Synthesis modeling and real-time compensation of geometric error and thermal error for CNC machine tools. *J. Mech. Eng.* **48**, 165–170,179.

[240] Yang J G, Ren Y Q, Zhu W B, Huang M L and Pan Z H. 2003. Research on on-line modeling method of thermal erroe compensation model for CNC machines. *J. Mech. Eng.* **39**, 81–84.

[241] Tseng P C. 1997. A real-time thermal inaccuracy compensation method on a machining centre. *Int. J. Adv. Manuf. Technol.* **13**, 182–190.

[242] Lin W Q, Fu J Z, Xu Y Z and Chen Z C. 2008. Thermal error modeling & compensation of numerical control machine tools based on on-line least squares support vector machine. *Comput. Integr. Manuf. Syst.* **14**, 295–299.

[243] Jin C, Wu B, Hu Y M, Yi P X and Cheng Y. 2015. Thermal characteristics of a CNC feed system under varying operating conditions. *Precis. Eng.* **42**, 151–164.

[244] Narendra K S and Mukhopadhyay S. 1997. Adaptive control using neural networks and approximate models. *IEEE Trans. Neural Netw.* **8**, 475–485.

[245] Chen J S, Yuan J X, Ni J and Wu S M. 1993. Real-time compensation for time-variant volumetric errors on a machining center. *J. Eng. Ind.* **115**, 472–479.

[246] Li Y L, Wang M Y, Hu Y M and Wu B. 2016. Thermal error prediction of the spindle using improved fuzzy-filtered neural networks. *Proc. Inst. Mech. Eng. B* **230**, 770–778.

[247] Wu Z, Ma C, Zhang L, Gui H Q, Liu J L and Liu Z J. 2024. Predicting and compensating for small-sample thermal information data in precision machine tools: a spatial-temporal interactive integration network and digital twin system approach. *Appl. Soft Comput.* **161**, 111760.

[248] Srivastava A K, Veldhuis S C and Elbestawit M A. 1995. Modelling geometric and thermal errors in a five-axis CNC machine tool. *Int. J. Mach. Tools Manuf.* **35**, 1321–1337.

[249] Feng W L, Li Z H, Gu Q Y and Yang J G. 2015. Thermally induced positioning error modelling and compensation based on thermal characteristic analysis. *Int. J. Mach. Tools Manuf.* **93**, 26–36.

[250] Ma C, Liu J L and Wang S L. 2020. Thermal error compensation of linear axis with fixed-fixed installation. *Int. J. Mech. Sci.* **175**, 105531.

[251] Liu K, Sun M J, Zhu T J, Wu Y L and Liu Y. 2016. Modeling and compensation for spindle's radial thermal drift error on a vertical machining center. *Int. J. Mach. Tools Manuf.* **105**, 58–67.

[252] Haitao Z, Jianguo Y and Jinhua S. 2007. Simulation of thermal behavior of a CNC machine tool spindle. *Int. J. Mach. Tools Manuf.* **47**, 1003–1010.

[253] Mian N S, Fletcher S, Longstaff A P and Myers A. 2013. Efficient estimation by FEA of machine tool distortion due to environmental temperature perturbations. *Precis. Eng.* **37**, 372–379.

[254] Mayr J, Ess M, Weikert S and Wegener K. 2009. Compensation of thermal effects on machine tools using a FDEM simulation approach. *Proc. Lamdamap* **9**, 38–48.

[255] Mayr J, Egeter M, Weikert S and Wegener K. 2015. Thermal error compensation of rotary axes and main spindles using cooling power as input parameter. *J. Manuf. Syst.* **37**, 542–549.

[256] Weng L T, Gao W G, Zhang D W, Huang T, Duan G L, Liu T, Zheng Y J and Shi K. 2023. Analytical modelling of transient thermal characteristics of precision machine tools and real-time active thermal control method. *Int. J. Mach. Tools Manuf.* **186**, 104003.

[257] Ruijun L, Wenhua Y, Zhang H H and Qifan Y. 2012. The thermal error optimization models for CNC machine tools. *Int. J. Adv. Manuf. Technol.* **63**, 1167–1176.

[258] Li Z M, Song Q H, Jin P J, Liu Z Q, Wang B and Ma H F. 2024. Chatter suppression techniques in milling processes: a state of the art review. *Chin. J. Aeronaut.* **37**, 1–23.

[259] Altintas Y and Weck M. 2004. Chatter stability of metal cutting and grinding. *CIRP Ann.* **53**, 619–642.

[260] Altintas Y and Ber A A. 2001. Manufacturing automation: metal cutting mechanics, machine tool vibrations, and CNC design. *Appl. Mech. Rev.* **54**, B84.

[261] Balamurugan S and Alwarsamy T. 2016. Machine tool chatter suppression techniques in boring operation-a review. *Asian J. Res. Soc. Sci. Human*. **6**, 2175–2188

[262] Leonesio M and Bianchi G. 2009. Manipulator calibration enhancement by means of mechanical templates. *Mech. Mach. Theory*. **44**, 1318–1320.

[263] Soori M. 2023. Deformation error compensation in 5-Axis milling operations of turbine blades. *J. Braz. Soc. Mech. Sci. Eng.* **45**, 289.

[264] Altintas Y, Brecher C, Weck M and Witt S. 2005. Virtual machine tool. *CIRP Ann.* **54**, 115–138.

[265] Abele E, Altintas Y and Brecher C. 2010. Machine tool spindle units. *CIRP Ann.* **59**, 781–802.

[266] Altintas Y, Verl A, Brecher C, Uriarte L and Pritschow G. 2011. Machine tool feed drives. *CIRP Ann.* **60**, 779–796.

[267] Altintas Y, Kersting P, Biermann D, Budak E, Denkena B and Lazoglu I. 2014. Virtual process systems for part machining operations. *CIRP Ann.* **63**, 585–605.
[268] Lu H J, Gao Z G, Sun Y N, Gao C J, Xu Z F, Pan Y J and Liu L L. 2025. Shape-performance coupled digital twin based on heterogeneous data from multiple sources: a scissor lift platform example. *Eng. Comput.* **41**, 609–626.
[269] Sun H, Peng F Y, Zhou L, Yan R and Zhao S Q. 2021. A hybrid driven approach to integrate surrogate model and Bayesian framework for the prediction of machining errors of thin-walled parts. *Int. J. Mech. Sci.* **192**, 106111.
[270] Tao F, Xiao B, Qi Q L, Cheng J F and Ji P. 2022. Digital twin modeling. *J. Manuf. Syst.* **64**, 372–389.
[271] Wei Y L, Hu T L, Zhou T T, Ye Y X and Luo W C. 2021. Consistency retention method for CNC machine tool digital twin model. *J. Manuf. Syst.* **58**, 313–322.
[272] Liu S M, Bao J S, Lu Y Q, Li J, Lu S Y and Sun X M. 2021. Digital twin modeling method based on biomimicry for machining aerospace components. *J. Manuf. Syst.* **58**, 180–195.
[273] Li Y F, Zhang Y J, Zhao Y Q and Shi X J. 2021. Thermal-mechanical coupling calculation method for deformation error of motorized spindle of machine tool. *Eng. Fail. Anal.* **128**, 105597.
[274] Saren Q, Zhang Z J, Xiong J, Chen X, Zhu D S, Wu W R, Jin X and Shang K. 2024. An accuracy and performance-oriented accurate digital twin modeling method for precision microstructures. *J. Intell. Manuf.* **35**, 2887–2911.
[275] Liu J L, Ma C, Gui H Q and Wang S L. 2022. A four-terminal-architecture cloud-edge-based digital twin system for thermal error control of key machining equipment in production lines. *Mech. Syst. Signal Process.* **166**, 108488.
[276] Huang Z W, Zhu J M, Lei J T, Li X R and Tian F Q. 2020. Tool wear predicting based on multi-domain feature fusion by deep convolutional neural network in milling operations. *J. Intell. Manuf.* **31**, 953–966.
[277] Zhang J, Deng C Y, Zheng P, Xu X and Ma Z T. 2021. Development of an edge computing-based cyber-physical machine tool. *Robot. Comput.-Integr. Manuf.* **67**, 102042.
[278] Xie Y, Lian K L, Liu Q, Zhang C Y and Liu H Q. 2021. Digital twin for cutting tool: modeling, application and service strategy. *J. Manuf. Syst.* **58**, 305–312.
[279] Liu J F, Wen X J, Zhou H G, Sheng S S, Zhao P, Liu X J, Kang C and Chen Y. 2022. Digital twin-enabled machining process modeling. *Adv. Eng. Inf.* **54**, 101737.
[280] Deng W K, Nguyen K T P, Medjaher K, Gogu C and Morio J. 2023. Physics-informed machine learning in prognostics and health management: state of the art and challenges. *Appl. Math. Model.* **124**, 325–352.
[281] Zio E. 2022. Prognostics and Health Management (PHM): where are we and where do we (need to) go in theory and practice. *Reliab. Eng. Syst. Saf.* **218**, 108119.
[282] Jiang T Y, Zhou J T, Luo X, Wang M W and Zhang S S. 2024. A systematic multi-layer cognitive model for intelligent machine tool. *J. Intell. Manuf.* **36**, 4915–4939.
[283] Osho J, Hyre A, Pantelidakis M, Ledford A, Harris G, Liu J and Mykoniatis K. 2022. Four Rs Framework for the development of a digital twin: the implementation of Representation with a FDM manufacturing machine. *J. Manuf. Syst.* **63**, 370–380.
[284] Wang J J, Li Y L, Gao R X and Zhang F L. 2022. Hybrid physics-based and data-driven models for smart manufacturing: modelling, simulation, and explainability. *J. Manuf. Syst.* **63**, 381–391.
[285] Mertes J, Glatt M, Schellenberger C, Klar M, Schotten H D and Aurich J C. 2022. Development of a 5G-enabled digital twin of a machine tool. *Proc. CIRP* **107**, 173–178.
[286] Schmid J, Schmid A, Pichler R and Haas F. 2020. Validation of machining operations by a Virtual Numerical Controller Kernel based simulation. *Proc. CIRP* **93**, 1478–1483.
[287] Tapia E, Lopez-Novoa U, Sastoque-Pinilla L and López-de-lacalle L N. 2024. Implementation of a scalable platform for real-time monitoring of machine tools. *Comput. Ind.* **155**, 104065.
[288] Tong X, Liu Q, Pi S W and Xiao Y. 2020. Real-time machining data application and service based on IMT digital twin. *J. Intell. Manuf.* **31**, 1113–1132.
[289] Tong X, Jiang H F, Ding G F, Lei J and Ma S W. 2021. Research on digital twin-based modeling and monitoring of five-axis grinder. *J. Syst. Simul.* **33**, 2880–2890.
[290] Altintas Y and Aslan D. 2017. Integration of virtual and on-line machining process control and monitoring. *CIRP Ann.* **66**, 349–352.
[291] Tan B, Zhang Y, Liu L B and Yang Z Q. 2020. Spindle system digital twin for dynamics characteristic monitoring. *China Mech. Eng.* **31**, 2231–2238,2246.
[292] Cai Y, Starly B, Cohen P and Lee Y S. 2017. Sensor data and information fusion to construct digital-twins virtual machine tools for cyber-physical manufacturing. *Proc. Manuf.* **10**, 1031–1042.
[293] Luo W C, Hu T L, Zhang C R and Wei Y L. 2019. Digital twin for CNC machine tool: modeling and using strategy. *J. Ambient Intell. Human Comput.* **10**, 1129–1140.
[294] Chou C H, Duan M L and Okwudire C E. 2021. A linear hybrid model for enhanced servo error pre-compensation of feed drives with unmodeled nonlinear dynamics. *CIRP Ann.* **70**, 301–304.
[295] Zhang J, Ding J X, Sugita N, Kizaki T, Li Q Z, Ding Q C and Wang L P. 2021. A sample construction method in kinematics characteristics domain to identify the feed drive model. *Precis. Eng.* **68**, 82–96.
[296] Jiang Y K, Chen J H, Zhou H C, Yang J Z and Xu G D. 2021. Residual learning of the dynamics model for feeding system modelling based on dynamic nonlinear correlate factor analysis. *Appl. Intell.* **51**, 5067–5080.
[297] Lee K F and Lee C H. 2020. Virtual feed drive system of machine tools development and applications in diagnosis and servo-tuning. *Asian J. Control.* **22**, 2167–2182.
[298] Wei Y L, Hu T L, Yue P J, Luo W C and Ma S H. 2024. Study on the construction theory of digital twin mechanism model for mechatronics equipment. *Int. J. Adv. Manuf. Technol.* **131**, 5383–5401.
[299] Jiang X L, Chen L Y, Zheng J J, Tan R Y, Li B Y and Fan D P. 2021. High-precision control method of direct drive components based on digital twin model. *J. Mech. Eng.* **57**, 98–109.
[300] Pi S W, Liu Q and Liu Q T. 2018. A novel dynamic contour error estimation and control in high-speed CNC. *Int. J. Adv. Manuf. Technol.* **96**, 547–560.
[301] Liang Z S, Wang S T, Peng Y L, Mao X Y, Yuan X, Yang A D and Yin L. 2022. The process correlation interaction construction of Digital Twin for dynamic characteristics of machine tool structures with multi-dimensional variables. *J. Manuf. Syst.* **63**, 78–94.
[302] Neugebauer R, Ihlenfeldt S, Hellmich A and Schlegel H. 2017. Modelling feed drives based on natural excitation-improving accuracy. *CIRP Ann.* **66**, 369–372.
[303] Cao X, Zhao G and Xiao W L. 2022. Digital Twin-oriented real-time cutting simulation for intelligent computer numerical control machining. *Proc. Inst. Mech. Eng. B* **236**, 5–15.
[304] Wang B C, Zhou H Y, Li X Y, Yang G, Zheng P, Song C, Yuan Y X, Wuest T, Yang H Y and Wang L H. 2024. Human Digital Twin in the context of Industry 5.0. *Robot. Comput.-Integr. Manuf.* **85**, 102626.

[305] Haag S and Anderl R. 2019. Automated generation of as-manufactured geometric representations for digital twins using STEP. *Proc. CIRP* **84**, 1082–1087.
[306] Yang Y, Yang Y, Xiao M Y, Wan M and Zhang W H. 2022. A Gaussian process regression-based surrogate model of the varying workpiece dynamics for chatter prediction in milling of thin-walled structures. *Int. J. Mech. Syst. Dyn.* **2**, 117–130.
[307] Dai Y N and Chen X B. 2018. Research on establishing prediction model for aerospace aluminum alloy milling force with the help of RBF neural network. *In Proceedings of the International Conference on Information Technology and Electrical Engineering 201* (ACM, Xiamen, China) p 20.
[308] Zheng Y Q, Tao T, Mei X S, Tian W W, Hu S and Du H Y. 2025. Modeling of thermal errors in electric spindle based on a hybrid approach of thermal deformation theory and data drive. *J. Intell. Manuf.* 1–17.
[309] Chen J L, Li S, Leng X L, Li C P, Kurniawan R, Kwak Y and Ko T J. 2023. Bionic digital brain realizing the digital twin-cutting process. *Robot. Comput.-Integr. Manuf.* **84**, 102591.
[310] Ghosh A K, Ullah A M M S, Teti R and Kubo A. 2021. Developing sensor signal-based digital twins for intelligent machine tools. *J. Ind. Inf. Integr.* **24**, 100242.
[311] Duan Z H, Wu T H, Guo S W, Shao T, Malekian R and Li Z X. 2018. Development and trend of condition monitoring and fault diagnosis of multi-sensors information fusion for rolling bearings: a review. *Int. J. Adv. Manuf. Technol.* **96**, 803–819.
[312] Kong L B, Peng X, Chen Y, Wang P and Xu M. 2020. Multi-sensor measurement and data fusion technology for manufacturing process monitoring: a literature review. *Int. J. Extrem. Manuf.* **2**, 022001.
[313] Hao Y P, Zhu L D, Qin S Q, Pei X Y, Yan T M, Qin Q Y, Lu H and Yan B L. 2024. On-machine inspection and compensation for thin-walled parts with sculptured surface considering cutting vibration and probe posture. *Int. J. Extrem. Manuf.* **6**, 065602.
[314] Tian C, Cho Y, Song Y, Park S, Kim I and Cho S Y. 2025. Integration of AI with artificial sensory systems for multidimensional intelligent augmentation. *Int. J. Extrem. Manuf.* **7**, 042002.
[315] Lu Y Q, Liu C, Wang K I K, Huang H Y and Xu X. 2020. Digital twin-driven smart manufacturing: connotation, reference model, applications and research issues. *Robot. Comput.-Integr. Manuf.* **61**, 101837.
[316] Liu Q, Leng J W, Yan D X, Zhang D, Wei L J, Yu A L, Zhao R L, Zhang H and Chen X. 2021. Digital twin-based designing of the configuration, motion, control, and optimization model of a flow-type smart manufacturing system. *J. Manuf. Syst.* **58**, 52–64.
[317] Li L H, Lei B B and Mao C L. 2022. Digital twin in smart manufacturing. *J. Ind. Inf. Integr.* **26**, 100289.
[318] Heo E and Yoo N. 2021. Numerical control machine optimization technologies through analysis of machining history data using digital twin. *Appl. Sci.* **11**, 3259.
[319] Deebak B D and Al-Turjman F. 2021. Digital-twin assisted: fault diagnosis using deep transfer learning for machining tool condition. *Int. J. Intell. Syst.* **37**, 10289–10316.
[320] Ji W, Yin S B and Wang L H. 2019. A big data analytics based machining optimisation approach. *J. Intell. Manuf.* **30**, 1483–1495.
[321] Vishnu V S, Varghese K G and Gurumoorthy B. 2023. A data-driven digital twin framework for key performance indicators in CNC machining processes. *Int. J. Comput. Integr. Manuf.* **36**, 1823–1841.
[322] He W W, Zhang L P, Hu Y, Zhou Z, Qiao Y S and Yu D. 2024. A hybrid-model-based CNC machining trajectory error prediction and compensation methtyleod. *Electronics* **13**, 1143.
[323] Zheng Y Q, Yang H B, Jiang G D, Hu S, Tao T and Mei X S. 2025. Data-mechanism fusion modeling and compensation for the spindle thermal error of machining center based on digital twin. *Measurement* **250**, 117152.
[324] Wang J J, Niu X T, Gao R X, Huang Z G and Xue R J. 2023. Digital twin-driven virtual commissioning of machine tool. *Robot. Comput.-Integr. Manuf.* **81**, 102499.
[325] Zhang Q, Zheng S G, Yu C J, Wang Q and Ke Y L. 2022. Digital thread-based modeling of digital twin framework for the aircraft assembly system. *J. Manuf. Syst.* **65**, 406–420.
[326] Zhao W K, Li R Y, Liu X L, Ni J, Wang C, Li C L and Zhao L B. 2023. Construction method of digital twin system for thin-walled workpiece machining error control based on analysis of machine tool dynamic characteristics. *Machines* **11**, 600.
[327] Jiang P Y, Jia F, Wang Y and Zheng M. 2014. Real-time quality monitoring and predicting model based on error propagation networks for multistage machining processes. *J. Intell. Manuf.* **25**, 521–538.
[328] Xiang S T and Altintas Y. 2016. Modeling and compensation of volumetric errors for five-axis machine tools. *Int. J. Mach. Tools Manuf.* **101**, 65–78.
[329] Lin M T and Wu S K. 2013. Modeling and analysis of servo dynamics errors on measuring paths of five-axis machine tools. *Int. J. Mach. Tools Manuf.* **66**, 1–14.
[330] Araghizad A E, Pashmforoush F, Tehranizadeh F, Kilic K and Budak E. 2024. Improving milling force predictions: a hybrid approach integrating physics-based simulation and machine learning for remarkable accuracy across diverse unseen materials and tool types. *J. Manuf. Process.* **114**, 92–107.
[331] Yang H J, Wang Z H, Zhang T and Du F X. 2020. A review on vibration analysis and control of machine tool feed drive systems. *Int. J. Adv. Manuf. Technol.* **107**, 503–525.
[332] Liu X, Li Y G, Cheng Y H and Cai Y. 2023. Sparse identification for ball-screw drives considering position-dependent dynamics and nonlinear friction. *Robot. Comput.-Integr. Manuf.* **81**, 102486.
[333] Lin Y N, Chen X, Zhang Q Y, You J Q, Xu R J, Wang Z R and Sun L F. 2025. Nano device fabrication for in-memory and in-sensor reservoir computing. *Int. J. Extrem. Manuf.* **7**, 012002.
[334] Hao J, Li C Y, Tan Z, Yao Z H and Sun W. 2025. Fully coupled thermal-structural-vibration characteristics of 3-axis machine tools with complex hybrid linear axis-spindle systems considering structural nonlinearities. *Mech. Syst. Signal Procress.* **224**, 112103.
[335] Guo Z Z, Liu S H and Wang H R. 2016. Dynamic modeling and experimental verification for the feeding system of a gantry machine tool based on thermal-mechanical coupling. *Int. J. Adv. Manuf. Technol.* **85**, 227–236.
[336] Li B R, Zhang H, Ye P Q and Wang J S. 2020. Trajectory smoothing method using reinforcement learning for computer numerical control machine tools. *Robot. Comput.-Integr. Manuf.* **61**, 101847.
[337] Wei X Y, Ye H H, Miao E M and Pan Q S. 2022. Thermal error modeling and compensation based on Gaussian process regression for CNC machine tools. *Precis. Eng.* **77**, 65–76.
[338] Yao J F, Yang Y, Wang X C and Zhang X P. 2023. Systematic review of digital twin technology and applications. *Vis. Comput. Ind. Biomed. Art.* **6**, 10.
[339] Xiaopeng Y, Teng H, Xiaohu W, Liang M and Guofu Y. 2023. Study on an approach for decoupling and separating the thermal positioning errors of machining center linear axes. *Int. J. Adv. Manuf. Technol.* **128**, 1139–1153.

[340] Altintas Y and Khoshdarregi M R. 2012. Contour error control of CNC machine tools with vibration avoidance. *CIRP Ann.* **61**, 335–338.
[341] Liang Y C, Li W D, Lou P and Hu J M. 2022. Thermal error prediction for heavy-duty CNC machines enabled by long short-term memory networks and fog-cloud architecture. *J. Manuf. Syst.* **62**, 950–963.
[342] Qi Q L and Tao F. 2018. Digital twin and big data towards smart manufacturing and industry 4.0: 360 degree comparison. *IEEE Access* **6**, 3585–3593.
[343] Wang H W, Ran Y, Zhang S Y and Li Y L. 2020. Coupling and decoupling measurement method of complete geometric errors for multi-axis machine tools. *Appl. Sci.* **10**, 2164.
[344] Shi H, Zhang B Y, Mei X S, Wang H T, Zhao F and Geng T. 2024. Thermal error modelling and compensation of CNC lathe feed system based on positioning error measurement and decoupling. *Measurement* **231**, 114633.
[345] Wang Z H, Wang S B, Wang S L, Liu N, Zhao Z Y and Wang Y K. 2023. A novel surface residual stress monitoring method based on the power consumption of machine tool: a case study in 5-axis machining. *J. Manuf. Process.* **86**, 221–236.
[346] Li J, Wang Z L, Zhang S Y, Ji J J, Xiang Y Z, Wang D T and Tan J R. 2025. Digital-twin virtual model real-time construction via spatio-temporal cascade reconstruction for full-field plastic deformation monitoring in metal tube bending manufacturing. *Robot. Comput.-Integr. Manuf.* **91**, 102860.
[347] Liu Z P, Lang Z Q, Gui Y F, Zhu Y P and Laalej H. 2024. Digital twin-based anomaly detection for real-time tool condition monitoring in machining. *J. Manuf. Syst.* **75**, 163–173.
[348] Cai E, Liu D, Liang L and Xu G H. 2015. Monitoring of chemical industrial processes using integrated complex network theory with PCA. *Chemom. Intell. Lab. Syst.* **140**, 22–35.
[349] Boukhechba K, Wu H Y and Bazine R. 2018. DCT-based preprocessing approach for ICA in hyperspectral data analysis. *Sensors* **18**, 1138.
[350] Yin L, Liu Q X, Zhang F, Zhou Z R and Ullah S. 2019. Study for the identification of dominant frequencies and sensitive structure on machine tools using modal decoupling and structural sensitivity analysis. *J. Vib. Eng. Technol.* **7**, 507–517.
[351] Li W, Feng Y Y, Zhang S H and Zuo W. 2021. Research on geometric error modelling and decoupling of machine tool based on NURBS projection curve. *Int. J. Adv. Manuf. Technol.* **117**, 2727–2740.
[352] Guo S J, Si Z X, Sa R, Zou Y H and Mei X S. 2024. Geometric error modeling and decoupling identification of rotary axis of five-axis machine tool based on spatial trajectory planning. *Measurement* **236**, 114887.
[353] Du H Y, Jiang G D, Tao T, Hou R S, Yan Z Z and Mei X S. 2023. A thermal error modeling method for CNC lathes based on thermal distortion decoupling and nonlinear programming. *Int. J. Adv. Manuf. Technol.* **128**, 2599–2612.
[354] Ren L M, Zhang W Q, Wang K K, Zhang P, Wang S Q and Tan Y S. 2022. Spindle-mounted self-decoupled force/torque sensor for cutting force detection in a precision machine tool. *Measurement* **204**, 112119.
[355] Luo M, Chong Z H and Liu D S. 2018. Cutting forces measurement for milling process by using working tables with integrated PVDF thin-film sensors. *Sensors* **18**, 4031.
[356] Xie Z H, Xie F G, Liu X J, Wang J S and Mei B. 2021. Tracking error prediction informed motion control of a parallel machine tool for high-performance machining. *Int. J. Mach. Tools Manuf.* **164**, 103714.
[357] Osei S, Wei W, Yu J H and Ding Q C. 2024. A new effective decoupling method to identify the tracking errors of the motion axes of the five-axis machine tools. *J. Intell. Manuf.* **35**, 3377–3392.
[358] Zhang Y Z, Zhang C X, Yan J, Yang C B and Liu Z F. 2022. Rapid construction method of equipment model for discrete manufacturing digital twin workshop system. *Robot. Comput.-Integr. Manuf.* **75**, 102309.
[359] Li M, Mao C H, Zhu Y, Yang K M and Li X. 2020. Data-based iterative dynamic decoupling control for precision MIMO motion systems. *IEEE Trans. Ind. Inf.* **16**, 1668–1676.
[360] Iñigo B, Colinas-Armijo N, de Lacalle L N L and Aguirre G. 2021. Digital twin-based analysis of volumetric error mapping procedures. *Precis. Eng.* **72**, 823–836.
[361] Liu J F, Cao X W, Zhou H G, Li L, Liu X J, Zhao P and Dong J W. 2021. A digital twin-driven approach towards traceability and dynamic control for processing quality. *Adv. Eng. Inf.* **50**, 101395.
[362] Li W, Cai Y J, Meng X Y, Zuo W and Miao K K. 2025. Spatial geometric error fusion and decoupling of multi-axis CNC machine tools based on Unscented Transform Kalman filter. *Measurement* **242**, 115826.
[363] Zhang H, Liu Q, Chen X, Zhang D and Leng J W. 2017. A digital twin-based approach for designing and multi-objective optimization of hollow glass production line. *IEEE Access* **5**, 26901–26911.
[364] Lu Q B, Zhu D, Wang M and Li M. 2023. Digital twin-driven thermal error prediction for CNC machine tool spindle. *Lubricants* **11**, 219.
[365] Chiu Y C, Wang P H and Hu Y C. 2021. The thermal error estimation of the machine tool spindle based on machine learning. *Machines* **9**, 184.
[366] Huo Z W, Sun Q J, Yu J R, Wei Y C, Wang Y F, Cho J H and Wang Z L. 2025. Neuromorphic devices assisted by machine learning algorithms. *Int. J. Extrem. Manuf.* **7**, 042007.
[367] Li B, Tian X T and Zhang M. 2019. Thermal error modeling of machine tool spindle based on the improved algorithm optimized BP neural network. *Int. J. Adv. Manuf. Technol.* **105**, 1497–1505.
[368] Cho S, Asfour S, Onar A and Kaundinya N. 2005. Tool breakage detection using support vector machine learning in a milling process. *Int. J. Mach. Tools Manuf.* **45**, 241–249.
[369] Wang J J, Xie J Y, Zhao R, Zhang L B and Duan L X. 2017. Multisensory fusion based virtual tool wear sensing for ubiquitous manufacturing. *Robot. Comput.-Integr. Manuf.* **45**, 47–58.
[370] Zhang J S and Zhang Y H. 2012. Research of CNC fault diagnosis based on RBF neural network. *Appl. Mech. Mater.* **150**, 211–216.
[371] Sundaram S, Puthenveetil F, Nair V S and Krishnaswamy R. 2024. The influence of material stiffness and damping on machining stability in boring tool–workpiece systems using finite element simulation to implement digital twin. *Int. J. Interact. Des. Manuf.* **18**, 6765–6784.
[372] Wang L P, Li W T and Yu G. 2022. Time domain study on the construction mechanism of milling stability lobe diagrams with multiple modes. *J. Manuf. Sci. Eng., Trans. ASME* **144**, 021007.
[373] Yu G, Wang L P, Wu J and Gao Y. 2020. Milling stability prediction of a hybrid machine tool considering low-frequency dynamic characteristics. *Mech. Syst. Signal Process.* **135**, 106364.
[374] Li W T, Wang L P, Yu G and Wang D. 2021. Time-varying dynamics updating method for chatter prediction in thin-walled part milling process. *Mech. Syst. Signal Process.* **159**, 107840.

[375] Chabeauti H, Ritou M, Lavisse B, Germain G and Charbonnier V. 2023. Digital twin of forged part to reduce distortion in machining. *CIRP Ann.* **72**, 77–80.
[376] Delbressine F L M, Florussen G H J, Schijvenaars L A and Schellekens P H J. 2006. Modelling thermomechanical behaviour of multi-axis machine tools. *Precis. Eng.* **30**, 47–53.
[377] Holkup T, Cao H, Kolář P, Altintas Y and Zelený J. 2010. Thermo-mechanical model of spindles. *CIRP Ann.* **59**, 365–368.
[378] Ramesh R, Mannan M A and Poo A N. 2003. Thermal error measurement and modelling in machine tools—part I. influence of varying operating conditions. *Int. J. Mach. Tools Manuf.* **43**, 391–404.
[379] Tai C Y and Altintas Y. 2023. A hybrid physics and data-driven model for spindle fault detection. *CIRP Ann.* **72**, 297–300.
[380] Ahmed Y S, Elmaraghy W and Elmaraghy H. 2024. Adaptive digital twin for product surface quality: supervisory controller for surface roughness control. *Int. J. Adv. Manuf. Technol.* **135**, 2117–2130.
[381] Booyse W, Wilke D N and Heyns S. 2020. Deep digital twins for detection, diagnostics and prognostics. *Mech. Syst. Signal Process.* **140**, 106612.
[382] Yang Q, Pattipati K R, Awasthi U and Bollas G M. 2022. Hybrid data-driven and model-informed online tool wear detection in milling machines. *J. Manuf. Syst.* **63**, 329–343.
[383] Liu J L, Ma C, Gui H Q and Li M Y. 2022. Geometric-thermal error control system for gear profile grinding machine. *Adv. Eng. Inf.* **52**, 101618.
[384] Gui H Q, Liu J L, Ma C, Li M Y and Wang S L. 2023. Mist-edge-fog-cloud computing system for geometric and thermal error prediction and compensation of worm gear machine tools based on ONT-GCN spatial-temporal model. *Mech. Syst. Signal Process.* **184**, 109682.
[385] Shi W S, Cao J, Zhang Q, Li Y H Z and Xu L Y. 2016. Edge computing: vision and challenges. *IEEE Internet Things J.* **3**, 637–646.
[386] Sun G Z, Fan K G and Yang J G. 2024. Digital-driven in-situ monitoring for thermally-induced volumetric errors of CNC machine tools. *J. Manuf. Process.* **131**, 2000–2015.
[387] Lin Y and Shen Y. 2003. Modelling of five-axis machine tool metrology models using the matrix summation approach. *Int. J. Adv. Manuf. Technol.* **21**, 243–248.
[388] Schwenke H, Franke M, Hannaford J and Kunzmann H. 2005. Error mapping of CMMs and machine tools by a single tracking interferometer. *CIRP Ann.* **54**, 475–478.
[389] Gao W, Haitjema H, Fang F Z, Leach R K, Cheung C F, Savio E and Linares J M. 2019. On-machine and in-process surface metrology for precision manufacturing. *CIRP Ann.* **68**, 843–866.
[390] Wang J, Zhao W Y, Leach R, Xu L, Lu W L and Liu X J. 2021. Positioning error calibration for two-dimensional precision stages via globally optimized image registration. *Measurement* **186**, 110222.
[391] Yeung H, Lane B M, Donmez M A and Moylan S. 2020. In-situ calibration of laser/galvo scanning system using dimensional reference artefacts. *CIRP Ann.* **69**, 441–444.
[392] Spur G, Hoffmann E, Paluncic Z, Benzinger K and Nymoen H. 1988. Thermal behaviour optimization of machine tools. *CIRP Ann.* **37**, 401–405.
[393] Koliskor A. 1971. Compensating for automatic-cycle machining errors. *Mach. Tool* **41**, 11–44.
[394] Donmez M A, Blomquist D S, Hocken R J, Liu C R and Barash M M. 1986. A general methodology for machine tool accuracy enhancement by error compensation. *Precis. Eng.* **8**, 187–196.
[395] Yang H and Ni J. 2003. Dynamic modeling for machine tool thermal error compensation. *J. Manuf. Sci. Eng.* **125**, 245–254.
[396] Moriwaki T. 1988. Thermal deformation and its on-line compensation of hydrostatically supported precision spindle. *CIRP Ann.* **37**, 393–396.
[397] Yan J Y and Yang J G. 2009. Application of synthetic grey correlation theory on thermal point optimization for machine tool thermal error compensation. *Int. J. Adv. Manuf. Technol.* **43**, 1124–1132.
[398] Miao E M, Liu Y, Liu H, Gao Z H and Li W. 2015. Study on the effects of changes in temperature-sensitive points on thermal error compensation model for CNC machine tool. *Int. J. Mach. Tools Manuf.* **97**, 50–59.
[399] Li Y X, Yang J G, Gelvis T and Li Y Y. 2008. Optimization of measuring points for machine tool thermal error based on grey system theory. *Int. J. Adv. Manuf. Technol.* **35**, 745–750.
[400] Mancisidor I, Beudaert X, Aguirre G, Barcena R and Munoa J. 2018. Development of an active damping system for structural chatter suppression in machining centers. *Int. J. Autom. Technol.* **12**, 642–649.
[401] Beudaert X, Erkorkmaz K and Munoa J. 2019. Portable damping system for chatter suppression on flexible workpieces. *CIRP Ann.* **68**, 423–426.
[402] Mori K, Kono D, Yamaji I and Matsubara A. 2017. Modelling of viscoelastic damper support for reduction in low frequency residual vibration in machine tools. *Precis. Eng.* **50**, 313–319.
[403] Dumanli A and Sencer B. 2021. Active control of high frequency chatter with machine tool feed drives in turning. *CIRP Ann.* **70**, 309–312.
[404] Chen Y L, Wang S, Shimizu Y, Ito S, Gao W and Ju B F. 2015. An in-process measurement method for repair of defective microstructures by using a fast tool servo with a force sensor. *Precis. Eng.* **39**, 134–142.
[405] Li X F, Zhao H, Zhao X and Ding H. 2016. Dual sliding mode contouring control with high accuracy contour error estimation for five-axis CNC machine tools. *Int. J. Mach. Tools Manuf.* **108**, 74–82.
[406] Dai J, Wan M, Qin X B and Zhang W H. 2023. A time-delay-based sliding mode controlling method for reducing contour errors of machine tools. *J. Manuf. Process.* **106**, 407–421.
[407] Guiassa R and Mayer J R R. 2011. Predictive compliance based model for compensation in multi-pass milling by on-machine probing. *CIRP Ann.* **60**, 391–394.
[408] Zhang D L and Chen H C. 2024. A data-driven iterative pre-compensation method of contouring error for five-axis machine tools. *Int. J. Adv. Manuf. Technol.* **135**, 1669–1684.
[409] Xu Z C, Zhang B L, Li D F, Yip W S and To S. 2024. Digital-twin-driven intelligent tracking error compensation of ultra-precision machining. *Mech. Syst. Signal Process.* **219**, 111630.
[410] Sun X M, Bao J S, Li J, Zhang Y M, Liu S M and Zhou B. 2020. A digital twin-driven approach for the assembly-commissioning of high precision products. *Robot. Comput.-Integr. Manuf.* **61**, 101839.
[411] Huang Z, Wei P X, Li C, Wang H Y and Wang J Y. 2020. Aero-engine blade profile reconstruction based on adaptive step size bat algorithm and visualization of machining error. *Proc. Inst. Mech. Eng. C* **234**, 49–65.
[412] Li G, Lu H X, Wang H, Ran Y C, Ji R J, Liu Y H, Zhang Y Z, Cai B P and Yin X K. 2025. A visualization method for cross-scale online monitoring of grinding state based on data-mechanism hybrid-driven digital twin system. *Mech. Syst. Signal Process.* **225**, 112293.
[413] Zhang W, Zhang X and Zhao W H. 2023. Research on the multi-physical coupling characteristics of the machine tool and milling process based on the systematically integrated model. *J. Manuf. Process.* **105**, 46–69.

[414] Zhang C X, Gao F, Che Y X and Li Y. 2015. Thermal error modeling of multisource information fusion in machine tools. *Int. J. Adv. Manuf. Technol.* **80**, 791–799.
[415] Liu K, Liu Y, Sun M J, Wu Y L and Zhu T J. 2016. Comprehensive thermal compensation of the servo axes of CNC machine tools. *Int. J. Adv. Manuf. Technol.* **85**, 2715–2728.
[416] Weixin H U, Cao Y L, Yang J X, Shang H C and Wang W B. 2018. An error prediction model of NC machining process considering multiple error sources. *Int. J. Adv. Manuf. Technol.* **94**, 1689–1698.
[417] Yang Z Q, Liu B B, Zhang Y R, Chen Y S, Zhao H W, Zhang G F, Yi W and Zhang Z H. 2024. Intelligent sensing of thermal error of CNC machine tool spindle based on multi-source information fusion. *Sensors* **24**, 3614.
[418] Wang L Q, Yang R J, Lv S S, Yang Z Q, Xin S W, Liu Y Q and Liu Q. 2025. Combined contour error control method for five-axis machine tools based on digital twin. *Sci. Rep.* **15**, 17809.
[419] Jiang H F, Qin S F, Fu J L, Zhang J and Ding G F. 2021. How to model and implement connections between physical and virtual models for digital twin application. *J. Manuf. Syst.* **58**, 36–51.
[420] Liu C Q, Li Y G and Shen W M. 2018. A real time machining error compensation method based on dynamic features for cutting force induced elastic deformation in flank milling. *Mach. Sci. Technol.* **22**, 766–786.
[421] Cao L, Khim G, Baek S G, Chung S C and Park C H. 2025. A selection method of key temperature points for thermal error modeling of machine tools featuring multiple heat sources. *Precis. Eng. Nanotechnol.* **93**, 528–539.
[422] Zhang L, Ma C and Liu J L. 2024. Enhancing four-axis machining center accuracy through interactive fusion of spatiotemporal graph convolutional networks and an error-controlled digital twin system. *J. Manuf. Process.* **112**, 14–31.
[423] Zhang L, Liu J H and Zhuang C B. 2024. Digital twin modeling enabled machine tool intelligence: a review. *Chin. J. Mech. Eng.* **37**, 47.
[424] Li M Y, Ma C, Liu J L, Gui H Q, Zeng S and Luo F Q. 2024. Thermal error prediction of precision boring machine tools based on extreme gradient boosting algorithm-improved sailed fish optimizer-bi-directional ordered neurons-long short-term memory neural network model and physical-edge-cloud system. *Eng. Appl. Artif. Intell.* **127**, 107278.
[425] Gui H Q, Liu J L, Ma C and Li M Y. 2024. Industrial-oriented machine learning big data framework for temporal-spatial error prediction and control with DTSMGCN model. *J. Intell. Manuf.* **35**, 1173–1196.
[426] Ma C, Zhao L, Mei X S, Shi H and Yang J. 2017. Thermal error compensation based on genetic algorithm and artificial neural network of the shaft in the high-speed spindle system. *Proc. Inst. Mech. Eng. B* **231**, 753–767.
[427] Ma C, Zhao L, Shi H, Mei X S and Yang J. 2017. Experimental and simulation study on the thermal characteristics of the high-speed spindle system. *Proc. Inst. Mech. Eng. C* **231**, 1072–1093.